THE INFLUENCE OF THE USE OF COMPUTERS IN THE TEACHING AND LEARNING OF FUNCTIONS IN SCHOOL MATHEMATICS

By

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Summary

The aim of the study was to investigate what influence the use of computers using MS Excel and RJS Graph software has on grade 11 Eritrean students' understanding of functions in the learning of mathematics. An empirical investigation using quantitative and qualitative research methods was carried out. A pre-test (task 1) and a post-test (task 2), a questionnaire and an interview schedule were used to collect data.

Two randomly selected sample groups (i.e. experimental and control groups) of students were involved in the study. The experimental group learned the concepts of functions, particularly quadratic functions using computers. The control group learned the same concepts through the traditional paper-pencil method.

The results indicated that the use of computers has a positive impact on students' understanding of functions as reflected in their achievement, problem-solving skills, motivation, attitude and the classroom environment.

Key words:

Technology, computers, functions, quadratic functions, graphs, school mathematics, mathematics performance.

Declaration

Student number: **3449-276-3**

I declare that **THE INFLUENCE OF THE USE OF COMPUTERS IN THE TEACHING AND LEARNING OF FUNCTIONS IN SCHOOL MATHEMATICS** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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Signature Date

(Zeslassie Melake Gebrekal)

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TABLE OF CONTENTS

Figures

Tables

CHAPTER 1

BACKGROUND AND OVERVIEW OF THE STUDY

1.1 Introduction

Paper-and-pencil manipulation has been the standard approach in the teaching and learning of mathematics for many years. However, technology has the potential to change that. Many traditional difficult problems can now be solved by pressing a few keystrokes, using the appropriate technology. Computers allow more powerful mathematical problem-solving and graphing opportunities in the learning and teaching of mathematics. They provide convenient, accurate and dynamic drawing, graphing and computational tools (NCTM, 2003) and give students opportunities to explore applications and concepts that would be too tedious and time consuming using paper-and-pencil techniques.

Nowadays, there is an increasing realisation that graphing technologies, particularly computers, may help secondary school students in learning mathematics and thus improve the ways of teaching and learning mathematics. Dunham and Dick (1994:444) note that graphing technologies have the potential to affect teaching and learning mathematics dramatically, particularly in the fundamental area of functions and graphs. Many authors have recommended the use of technology at all levels of mathematics instruction (Dessart, DeRidder & Ellington, 1999; Fey, 1989; NCTM, 2000, 2003; President's Committee of Advisors on Science and Technology, 1997; Taylor, 1980). *The Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) advocate extensive use of computers to transform the mathematics curriculum. At present, there is also a growing body of research related to the use of computers in the mathematics classroom (Battista, 2001; Connell, 2001; De Villiers, 1999, 2004; Ibrahim, 2004; Hannafin & Scott, 2001; Hennessy, Fung & Scanlon, 2001; Liu & Cummings, 2001; Tooke, 2001; Wiest, 2001). Researchers worldwide have been striving for better ways of teaching and learning mathematics by integrating new strategies with technology mediated instruction.

The focus of this study is, therefore, on the use of computers using MS Excel and RJS Graph software in the teaching and learning of mathematics (functions in particular).

1.2 Statement of the Problem

In Eritrea, the structure of the existing system of education is 5 - 3 - 4. The first five years for elementary school, grades 6 - 8 for middle school and grades 9 - 12 for secondary school. Mathematics is a compulsory subject at all levels from elementary to secondary school.

The notion of function is greatly emphasized in the secondary school mathematics curriculum in Eritrea. As indicated in the curriculum framework for secondary level mathematics (Ministry of Education, 2005), concepts of functions are provided in grades 9, 11 and 12. However, from my experience, concepts of functions are among the difficult topics in the teaching and learning of mathematics.

During my teaching of Mathematics, it became evident that there are a number of conceptual obstacles to progress in concept formulation regarding functions. One of the obstacles is the difficulty to construct graphs of functions. In order to construct graphs of functions using paper and pencil, students spend a lot of time in performing repeated algorithmic computations and sketching the graphs (to find and plot points) which is a tedious work (Confrey, 1992:151; Fey, 1989:249; Rich, 1993:389). In this case, they do not get sufficient time to explore the nature and properties of functions and their graphs.

The construction of graphs of functions using paper and pencil has not only hindered students' progress in understanding functions but has also fostered in students a negative attitude towards mathematics in general and towards functions in particular.

The development of technology, however, is a promising prospect to ease the difficulty of teaching functions and to improve students' learning of functions. Computer technology allows students to graph functions more easily, quickly and accurately; to manipulate the graphs; and to develop generalizations about the functions. It also allows students to form linked multiple-representations of mathematical concepts (Heid, 1998; Waits & Demana, 2000) and to explore, estimate and discover them graphically and to approach problems from a multi representational perspective (Hennessy *et al.*,2001:283; Hollar & Norwood, 1999:222). Beckmann, Senk and Thompson (1999:451), Confrey (1992:150) and Fey (1989:255) point out that computer technology offers students the opportunity to explore the concepts and notion of functions in multi-representational (symbolic, numeric, tabular and graphic or visual) modes. In addition to this, Nicaise and Barnes (1996:208) mention that "Once adept at using technology, students have quick access to multiple resources and tools for combining those resources. They can spend less time looking for answers and information and more time analyzing, reflecting, and developing an understanding." Furthermore, Fey (1989:240) notes, "When used wisely, technology can enhance student conceptual understanding, problem solving and attitudes toward mathematics." Dunham and Dick (1994:443) also observe that students can improve their problem-solving abilities and attitudes when they use graphing technology.

The study therefore seeks answers to the following question:

What is the influence of the use of computers using MS Excel and RJS Graph software on grade 11 Eritrean students' understanding of functions in school mathematics?

1.3 Aim of the Study

The aim of this study is to investigate what influence the use of computers using MS Excel and RJS Graph software has on grade 11 Eritrean students' understanding of functions in the learning of mathematics.

1.4 Research Design and Methodology

In order to answer the research question raised in paragraph 1.2, I will use an eclectic design according to the elements of triangulation. De Vos (2002:342, 365) describes triangulation as mixing qualitative and quantitative styles of research and data. Ngwenya (in Makgato 2003:11) and Jick (1994:191) define triangulation as the use of two or more methods such as tasks, questionnaires, interviews and observations in the study of human behaviour. Ngwenya (in Makgato, 2003:11) further mentions that the purpose of using triangulation is to explain fully the richness and complexity of human behaviour by studying it from more than one viewpoint.

The use of one method of data collection tends to be biased and distorts the researcher's picture of the particular slice of reality under investigation (Cohen & Manion, 1980:208). In applying the triangulation techniques in this study, the following research design and data collection methods will be used.

1.4.1 Literature Study

A literature study involves the systematic identification and analysis of documents containing information related to the research problem (Gay, 1992:38). It is a process of gathering information regarding the research problem and the current state of knowledge on the topic to be investigated (Sax, 1979:53).

Neuman (1997:89) asserts that literature helps the researcher to:

- demonstrate a familiarity with a body of knowledge and to establish credibility;
- show the path of prior research and how a current project is linked to it;
- integrate and summarize what is known in an area; and
- learn from others and stimulate new ideas.

I, therefore, will have to consult a wide variety of sources such as publications, books, journals, newspaper articles, dissertations and theses which are relevant to the study. By consulting these sources, I will gain theoretical knowledge on the influence of computers in mathematics instruction.

1.4.2 Empirical Investigation

In the field study, both quantitative and qualitative research methods will be used. First, an educational experiment will be conducted to ascertain whether the use of computers using MS Excel and RJS Graph software has a significant effect on students' understanding of functions (achievements). Consequently, the results obtained will be enriched by implementing quantitative and qualitative research processes in which respectively questionnaires and interviews will be utilized.

1.5 Clarification of Concepts

1.5.1 Functions

Concepts of functions are considered to be amongst the most important concepts in mathematics for students to master. The topic of functions, as in the mathematics curriculum, focuses on correspondence relationships. Posthuma (2000:22) defines a function as a relationship between dependent and independent variables (quantities) such that, for each value of the independent variable there corresponds exactly one value of the dependent variable and it can be represented:

- by means of an equation;
- in a table form;
- as a piecewise-defined function; or
- by means of a graph.

When one quantity is described as a function of another, it means that the first quantity depends in some way on the second.

In the secondary school mathematics curriculum we find various types of functions such as linear function, quadratic function, exponential function and logarithmic function. The focus of this study is on quadratic function (in the form of $f(x) = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$).

1.5.2 The Use of Computers

This refers to students' use of computers (MS Excel and RJS Graph software) in learning functions in school mathematics. The students can construct graphs of functions and do exploration of mathematical ideas using computers.

1.6 Layout of the Study

Chapter 1 provides an introduction, statement of the problem, the aim of the study and method of data collection.

Chapter 2 gives a review of the literature study that was done.

Chapter 3 deals with the research design as well as the methods and techniques (instruments) used for the data collection.

Chapter 4 provides the presentation, analysis and interpretation of the data.

Chapter 5 consists of a summary of the findings followed by conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to present a literature review on the influence of the use of technology in the teaching and learning of mathematics. The focus will be on the use of the computer as technological tool. A review of studies related to its use and the influence thereof in the teaching and learning of mathematics will be reported. Theories of and approaches to mathematics learning and teaching and the function concept are also discussed. The following section discusses theories of and approaches to mathematics learning and teaching.

2.2 Theories of and Approaches to Mathematics Learning and Teaching

For many years mathematics was taught in what is referred to as the traditional way with the teacher transmitting all the knowledge and the child passively accepting it without question (Nariansamy in Ijeh, 2003:35). In the traditional mathematics classroom, where the teacher only shows how and what is to be done, there is little discussion; pupils are seldom given chance to ask questions if they do not understand something. Often children, who already built up a fear of mathematics, feel afraid of the teacher and the reaction of peers if they do not understand (Nariansamy in Ijeh, 2003:38).

On the other hand, a mathematics classroom where meaningful teaching and learning takes place provides a powerful means of communication between the teacher and the student or among the students themselves. In contrast, the traditional mathematics classroom is ironically a place where the children's opinions are never heard (Nariansamy in Ijeh, 2003:36).

Since 1980, however, the theory of constructivism has been advocated as an effective way of learning and teaching mathematics. According to this theory, learners actively construct their own knowledge with the focus on a problem-centered approach based on constructivist perspectives. Constructivists believe that learning is the discovery and transformation of complex information and that traditional teacher-centered instruction of predetermined plans, skills and content is inappropriate (Nicaise & Barnes, 1996:206). Furthermore, they suggest that situations and social activities shape understanding. They are critical of traditional teachers when they do not provide students with essential contextual features of learning, thus forcing students to rely on superficial, surface-level features of problems without the abilities to apply or use knowledge. Nicaise and Barnes (1996:206) suggest that learning occurs within the world students experience and that when they deal with problems and situations simulating and representing authenticity, they learn more. The following section discusses constructivist learning theories and problem-solving and problem-centered approaches to teaching and learning mathematics.

2.2.1 A Constructivist Perspective on Teaching and Learning

Constructivism is an epistemology that views knowledge as being constructed by learners from their prior experience. The learner interacts with his/her environment and thus gains an understanding of its features and characteristics. The learner constructs his/her own conceptualisations and finds his/her own solutions to problems, mastering autonomy and independence. According to constructivism, learning is the result of individual mental construction, whereby the learner learns by dint of matching new against given information and establishing meaningful connections, rather than by internalising mere factoids to be regurgitated later on (Thanasoulas, 2002, http://www3.telus.net/linguisticsissues/constructivist.html). Thanasoulas (2002, http://www3.telus.net/linguisticsissues/constructivist.html) notes that in constructivist thinking, learning is inescapably affected by the context and the beliefs and attitudes of the learner. Here, learners are given more latitude in becoming effective problem solvers, identifying and evaluating problems, as well as deciphering ways in which to transfer their learning to these problems.

Constructivist learning is based on students' active participation in problem-solving and critical thinking regarding a learning activity that they find relevant and engaging. They are "constructing" their own knowledge by testing ideas and approaches based on their prior knowledge and experience, applying these to a new situation and integrating the new knowledge gained with pre-existing intellectual constructs. In this view, knowledge is gained by an active process of construction rather than by passive assimilation of information or rote memorization. This view of learning sharply contrasts with one in which learning is the passive transmission of information from one individual (teacher) to another (student), a view in which reception, not construction, is the key.

According to constructivist learning theory, mathematical knowledge cannot be transferred ready-made from one person (teacher) to another (student). It ought to be constructed by every individual learner. This theory maintains that students are active meaning-makers who continually construct their own meanings of ideas communicated to them. This is done in terms of their own existing knowledge base. This suggests that a student finds a new mathematical idea meaningful to the extent that he/she is able to form a new concept (Bezuidenhout, 1998:390).

Kamii (1994:21) states that “Children have to go through a constructive process similar to our ancestors’, at least in part, if they are to understand today’s mathematics.” Kamii (1994:32) goes on to say that, today’s mathematics are the results of centuries of construction by adult mathematicians. By trying to transmit in a ready-made form the results of centuries of reflection by adults, we deprive children of opportunities to do their own thinking. Students today invent the same kinds of procedures our ancestors did and need to go through a similar process of construction to become able to understand adults’ mathematics.

Students’ first methods (algorithms) are admittedly inefficient. However, if they are free to do their own thinking, they invent increasingly efficient procedures just as our ancestors did. By trying to bypass the constructive process, we prevent them from making sense of mathematics.

Reys, Suydam, Lindquist and Smith (1998:19) mention three basic tenets on which constructivism rests. These are:

- Knowledge is not passively received; rather, knowledge is actively created or invented (constructed) by students.
- Students create (construct) new mathematical knowledge by reflecting on their physical and mental activities.
- Learning reflects a social process in which children engage in dialogue and discussion with themselves as well as others (including teachers) as they develop intellectually.

There are three types of constructivism that are applicable to mathematics education. They are known as:

- **Radical constructivism:** According to this theory, knowledge cannot simply be transferred ready-made from parent to child or from teacher to student but has to be actively built by each learner in his/her own mind (Glasersfeld, 1992). This implies that students usually deal with meanings, and when instructional programs fail to develop appropriate meanings, students create their own meanings. Ernest (1991) observes this type of constructivism lacks a social dimension in which the students learn dependently. Cobb, Yackel and Wood (1992:27) also contend that “the suggestion that students can be left to their own devices to construct the mathematical ways of knowing compatible with those of wider society is a contradiction of terms.”
- **Social-constructivism:** Ernest (1991) comes up with a new type of constructivism that is called social-constructivism which views mathematics as a social construction which means that students can better construct their knowledge when it is embedded in a social process. Through the use of language and social interchange (i.e. negotiation between the teacher and the students and

among the students), individual knowledge (understanding) can be expressed, developed and contested.

- **Socio-constructivism:** This type of constructivism is developed only in mathematics education. According to this theory, mathematics is a creative human activity and mathematical learning occurs as students develop effective ways to solve problems. In connection with this, Jones (1997:145) notes,

> Knowledge is the dynamic product of the work of individuals operating in the communities, not a solid body of immutable facts and procedures independent of mathematicians. In this view, learning is considered more as a matter of meaning-making and of constructing one's own knowledge than of memorizing mathematical results and absorbing facts from the teacher's mind or the textbook; teaching is the facilitation of knowledge construction and not delivery of information.

Supporters of socio-constructivism theory claim that when individuals (learners as well as the teacher) interact with one another in the classroom, they share their views and experiences and along the way knowledge is constructed. Knowledge is acquired through the sharing of their experiences. Therefore, it is socially constructed (Ernest, 1991; Stein, Silver & Smith, 1998).

Vygotsky holds the anti-realist position that the process of knowing is rather a disjunctive one involving the agency of other people and mediated by community and culture. He sees collaborative action to be shaped in childhood when the convergence of speech and practical activity occurs and entails the instrumental use of social speech. Although in adulthood social speech is internalized (it becomes thought), Vygotsky contends, it still preserves its intrinsic collaborative character (Kanselaar, 2002).

Vygotsky (in Nicaise and Barnes, 1996:207) articulated the importance of social discourse when he suggested that cognitive development depends on the child's social interaction with others, where *language* plays a central role in cognition. Vygotsky believes that social interaction guides students thinking and concept formation (schema). Conceptual growth occurs when students and teachers share different viewpoints and experiences and understanding changes in response to new perspectives and experiences (Nicaise & Barnes, 1996:207).

The characteristics of socio-constructivism are:

i) mathematics should be taught through problem-solving;
ii) students should interact with teachers and other students as well; and
iii) students are stimulated to solve problems based on their own strategies (Cobb *et al.*,1992).

2.2.2 Problem-solving and Problem-centered Approaches to Teaching and Learning Mathematics

a) Problem-solving Approach

A problem-solving approach is an approach to teaching mathematics. With this approach the focus is on teaching mathematical topics through problem-solving contexts and enquiry-oriented environments which are characterized by the teacher helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing and verifying (Lester, Masingila, Mau, Lambdin, dos Santon & Raymond in Taplin, 2007). According to Taplin's (2007) review of research reports, specific characteristics of a problem-solving approach include:

- interactions between students mutually as well as teachers and students;
- mathematical dialogue and consensus between students;
- teachers providing just enough information to establish background/intent of the problem and students clarifying, interpreting and attempting to construct one or more solution processes;
- teachers accepting right/wrong answers in a non-evaluative way;
- teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems;
- teachers knowing when it is appropriate to intervene and when to step back and let the pupils make their own way; and

- the possibility of using such an approach to encourage students to make generalizations about rules and concepts, a process which is central to mathematics.

b) Problem-centered Approach

A problem-centered approach is also an approach to mathematics education that is based on problem-solving. We could just as easily have called this a *learner-centered* approach or, to use the more formal term, *constructivist;* it follows the theory that learning occurs when students construct their own knowledge. In problem-centered mathematics instruction, students construct their own understanding of mathematics through solving reality-based problems, presenting their solutions and learning from one another's methods. The learner interprets the problem conditions in the light of his/her repertoire of experiences (knowledge and strategies previously assimilated). The teacher provides the necessary scaffolding during this process.

Problem-centered approach theory opposes the view that mathematics is a ready-made system of rules and procedures to be learned; a static body of knowledge. According to this theory, mathematics is a human activity and students must engage in a way similar to the genetic development of the subject. Supporters of this theory hold that students should not be considered as passive recipients of ready-made mathematics, but rather that education should guide the students towards using opportunities to invent (re-invent) mathematics by doing it themselves (Ndlovu, 2004:19). Students should be given the opportunity to experience their mathematical knowledge as the product of their own mathematical activity.

In a problem-centered approach, instruction begins with reality-based problems, dilemmas and open-ended questions. The learners acquire knowledge from the solution of problems. They engage in a variety of problem situations and along the process learn mathematical content (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne, 1996:19). They also use mathematical knowledge to solve real life problems.

- **The Role of Social Interaction**

The problem-centered classroom is a place where problem posing and problem-solving takes place. These processes are characterized by invention, explanation, negotiation, sharing and evaluation (Nakin, 2003:65). As Murray, Olivier & Human (1993:194) point out in this regard, social interaction creates the opportunity for children to talk about their thinking and encourages reflection; students learn not only from their own constructions but also from one another and through interaction with the teacher. The opportunity to exchange, discuss and evaluate one's own ideas and the ideas of others encourages decentration (the diminution of egocentricity), thereby leading to a more critical and realistic view of the self and others (Piaget in Post, 1980:115).

- **The Role of the Teacher**

In a problem-centered classroom, the role of the teacher is no longer that of transmitter of knowledge to students, but rather a facilitator of their learning. He/she has "the role of selecting and posing appropriate sequences of problems as opportunities for learning, of sharing information when it is necessary for tackling problems, and of facilitating the establishment of a classroom culture in which pupils work on novel problems individually and interactively, and discuss and reflect on their own answers and methods" (Hiebert, Carpenter, Fennema, Fusson, Human, Murray, Olivier & Wearne, 1997:8). Casey (1997:79) compares traditional views with current views on the roles of teachers and learners in learning as follows:

> The old teaching paradigm implies that learning only happens when the teacher puts information into children's heads. The new paradigm implies that children can construct and learn … when they are in control. The new paradigm does not imply that the teacher is unnecessary … a knowledgeable teacher who acts as a guide, facilitator, or fellow learner is essential.

Teachers have to constantly assist and support individual learners to develop their cognition at their own level and pace. The teacher has to plan, set up, manage and evaluate the teaching and learning activities to benefit the total development of every individual in the classroom. He/she must be thoroughly organized in planning appropriate activities, providing opportunities and creating a classroom atmosphere with his/her learning objectives in mind. He/she should create learning environments containing multiple sources of information and multiple viewpoints where students think, explore and construct meaning (Nicaise & Barnes, 1996:207) and situations to develop creative thinking and develop a wide range of problem-centered activities and materials to aid problem-solving development in learners. He/she should also encourage learners to think critically, adapt ideas that make sense to them, invent many different ways to solve problems and to expand and enhance the development of mathematical concepts through problem-solving activities.

Teachers guide learners to discover and develop mathematics skills, such as active inquiry and reflection, in order to analyze and synthesize information, solve problems and successfully construct new knowledge through creative participation and understanding. Progressive teachers facilitate learning by selecting and implementing suitable learning matter and by motivating learners to improve their personal skills and abilities through the use of different materials and tools, such as computers. Teachers observe and evaluate learners' progress and provide them with relevant feedback in this regard. They thus monitor and guide rather than dominate and direct learning activities (Bonk & King, 1998:370; Newby, Stepich & Russel, 2000:146).

- **The Role of the Learners**

In the problem-centered approach, learners choose and share their methods (Hiebert *et al*., 1997:9). Learners should also be free to express themselves without fear of reprisal. Mistakes are often as constructive as the correct strategies in helping learners to understand the mathematics involved (Erickson, 1999:518; Hiebert *et al*., 1997:9). According to Hiebert *et al*. (1997:9), mistakes provide opportunities for examining errors in reasoning, and thereby raise learners' level of analysis. Learners should realise that learning means learning from others and must take advantage of others' ideas and results of their investigations.

2.3 The Function Concept

The function concept is a mathematical representation of many input-output situations found in the real world (NCTM, 1989:154) and plays a central and fundamental role in mathematics. The notion of functions is greatly emphasised in secondary school mathematics. However, the function concept has proved to be problematic for students to master. Some of these difficulties are:

- *The concept of variable*: Students often misunderstood the concept of a variable and how variables allow them to construct mathematical meanings (Graham & Thomas, 2000). Even though variables are fundamental to functional relationships and graphical representations (Leinhardt, Zaslavsky & Stein, 1990), students find it very difficult to understand functions and their graphs as abstractions.
- *The process-object duality*: A mathematical concept often has two faces: an operational process side and a structural object side. For students, the process aspect initially dominates the concept. A mature understanding, however, includes the object side and the flexibility to switch between the two views. The flexibility to shift between the process and the object perspective is indispensable for mature mathematical thinking (Gray & Tall, 1994; Tall & Thomas, 1991).
- *Connections*: Students have difficulties in making connections between different representations of the notion (tables, equations, graphs, diagrams and word descriptions) (Elia & Spyrou, 2006:257; Ernest, 1989:38)
- *Interpretation:* Students have difficulties in interpreting graphs of functions (Elia & Spyrou, 2006:257; Zaslavsky, 1997:30). Zaslavsky (1997:31) notes that the graphs of a quadratic function may seem as if it is limited only to the visible part that is actually drawn, although, in fact, it represents an infinite domain.

Numerous research studies, however, note that graphing technologies offer much promise to facilitate the learning of functions (for example, Leinhardt *et al.,* 1990;

Zaslavsky, 1997). Zaslavsky (1997:36) points out that the availability and use of graphical technologies such as computers is important in overcoming the difficulties with developing students' conceptual understandings regarding functions. Confrey (1992:151) observes that with the development of graphical interfaces that are both fast and flexible, computers offer an exciting setting for teaching functions. Beckmann *et al.* (1999:451) note that technological tools allow the investigation of functions through tables, graphs and equations in ways that were not possible before their proliferation. Ernest (1989:38) and Sutherland (1990:164) observe that LOGO provides a meaningful context for the introduction of a variable concept. Fey (1989:255) also suggests that technology provides possibilities for multiple representation of the notion of functions which help students to develop a conceptual understanding of functions.

2.4 Technology and Mathematics Education

Technological tools such as computers and calculators can revolutionalise the existing traditional mathematics instruction by providing more powerful mathematical problem-solving and graphing opportunities and offering new possibilities in the learning and teaching of mathematics (see Fey, 1989; Heid, 1998; Hennessy *et al.,* 2001). Mathematics education is considering technology as a catalyst for change (Dunham & Dick, 1994:440; Heid, 1997:5; Heid, 1998). In light of the technological opportunities, it has often been suggested that these tools greatly influence mathematics education (Fey, 1989; Heid, 1998). Beckmann *et al.* (1999:451) and Fey (1989:237) observe that these technological tools challenge mathematical education in terms of ‘what we should teach’, ‘how we should teach’, ‘what students can learn’ and 'how students can learn it'.

2.4.1The Role of Technology in Teaching and Learning Mathematics

The great potential of computer technologies in mathematics instruction is increasingly believed to bring a transformation in mathematics education and has brought new possibilities to the teaching and learning of mathematics. Goldenberg (2000:1) points out that one of the strongest forces in the contemporary growth and evolution of mathematics and mathematics teaching is the power of new technologies. Goldenberg (2000:1), thereupon, claims that "In math, computers have fostered entirely new fields. In education, they have raised the importance of certain ideas, made some problems and topics more accessible, and provide new ways to represent and handle mathematical information, affording choices about content and pedagogy that we have never had before."

Technological tools such as computers and calculators can play a significant role in providing an environment where students may gain appropriate experiences to construct mathematical concepts. In an environment where technologies are available there is a shift in the emphasis of mathematics instruction since the algorithmic computations involved in traditional mathematics instruction are often lengthy and time-consuming. Most of the class time is devoted to rote practice of these procedures. But in a technologically rich classroom environment the instruction can change to concept development and problem-solving by concentrating on the underlying concepts since these tools remove the burden of lengthy and time-consuming routine work (Branca, Breedlove & King, 1992; Fey, 1989; Hennessy *et al.,* 2001; Wheatley & Shumway, 1992).

Technological tools allow students to graph functions more easily, quickly and accurately; to manipulate the graphs; and to develop generalizations about the functions. More time can be spent on analyzing the graphs and less time on the actual development of the graphs. Students build deeper understanding of functions and the graphs of the functions since less time is spent performing calculations. Pomerantz (in Dreiling, 2007:2) notes that "By reducing the time that, in the past, was spent on learning and performing tedious paper-and-pencil arithmetic and algebraic algorithms, calculator use today allows students and teachers to spend more time developing mathematical understanding, reasoning, number sense, and applications."

Dick (in Dunham and Dick, 1994:442-443) mentions three ways in which graphing tools can lead to the improvement of problem-solving: (1) they free more time for instruction by reducing attention to algebraic manipulation; (2) they supply more tools for problem-solving and can serve as a monitoring aid during the problem-solving process; and (3) students perceive problem-solving differently when they are freed from the burden of numerical and algebraic computation to concentrate on setting up the problem and analyzing the solution. Computers can be used most effectively to help students gather data and test, modify and reject or accept conjectures as they think about these mathematical concepts and experience mathematical research (Cuoco & Goldenberg, 1996:1). Fey (1989:255-256) notes that computer technology "plays a role in helping move students from concrete thinking about an idea or a procedure to the ultimately more powerful abstract symbolic form…It plays a role as a kind of intermediate abstraction."

Technology brings a new richness of information into the classroom and provides students with access to multiple sources of information that could be used to solve complex problems. Sound, pictures, video, graphs, charts, maps, three-dimension and animation all make for interesting, exciting content. Advances in the processing power of computers permit students to visualize phenomena formerly invisible and to instantly grasp relationships once obscure or difficult to understand. Pictures and graphics add new dimensions to ways of presenting information that was responsive to alternative learning styles.

Using computer technology, pupils can transform a graph and watch the algebraic symbolism change or alternatively manipulate the symbolism and watch the graphical representation changes (Sutherland, 1990:168). Kaput (in Sutherland, 1990:168) claims that:

> ... the dynamic nature of the medium supports dynamic changes in variable values that renders the underlying ideas of variable and function more learnable, which should make them accessible to a younger population, and which in return makes possible a much more gradual and extended algebra curriculum, beginning in the early grades.

Technology can enable students to explore relevant mathematical ideas through constructivist methods (Pugalee, 2001). It serves students as an information resource, a learning tool or a storage device that can support students to construct their own mathematical knowledge (Nicaise & Barnes, 1996:207) and allows students to actively participate and be responsible for their own learning. Technology supports exploration, which helps students set achievable goals, form and test hypotheses and makes discoveries of their own (Collins, 1991). In an environment where technologies are available, students might be involved in running experiments, testing conjectures, solving and posing problems and exchanging ideas (Heid, 1998). In connection with this, Lewis (1999:142) writes, "Constructive learning stresses active, outcome-orientated and self-regulated learning, where meaning is negotiated and multiple perspectives are encouraged. The flexible interactive characteristics of computer technologies are enormously supportive of this." Thus, the availability of technologies in school mathematics may allow students to explore mathematics on their own.

Computer technology offers students varieties of linked approaches to the same problem situation. It allows students to form linked multiple-representations of mathematical concepts (Heid, 1998; Waits & Demana, 2000) and to explore, estimate and discover them graphically and to approach problems from a multi-representational perspective (Hennessy *et al*., 2001:283; Hollar & Norwood, 1999:222). Beckmann *et al.* (1999:451), Confrey (1992:150), Fey (1989:255) and Heid (1998) suggest that computer technology offers students the opportunity to explore the concepts and notion of functions in multi-representational (symbolic, numeric, tabular and graphic or visual) modes. Computer technology also helps students to make connections between mathematical ideas (Smith & Shotsberger, 1997), between a real world phenomenon and its mathematical representations and between a student's everyday world and his/her mathematical world (Heid, 1998).

Dick, Wilson and Krapfl (in Beckmann *et al.*,1999:451), Heid (1998) and Hennessy *et al*. (2001: 283) suggest that the use of multiple representations, interpretation from one representation to another, and the analysis requiring interplay between graphic, numeric and symbolic information are keys to understanding functions. A student who makes connections between mathematical ideas creates a deeper understanding of those ideas and different representations of a problem allow a student to represent the problem in a way that best makes sense to the student (NCTM, 2000). Furthermore, Davidenko (1997:149) notes that the use of computer technology is "ideal to promote connections between mathematics and students' everyday experiences, to develop mathematical language and reasoning skills, and to create a cooperative environment in mathematics classroom."

One characteristic of technology-mediated instruction is interactivity. The availability of technological tools in mathematics instruction plays a role in facilitating interactions and cooperative group work among students and teachers (Heid, 1997; Heid, 1998; Hennessy *et al*., 2001; Nicaise and Barnes, 1996). Technological tools provide an area that is rich in social interaction and facilitate students' communication with other students through formal presentations, cooperative activities, collaborative problem-solving and interpersonal exchanges. Students can experience enjoyment and surprise and develop interest as they explore software, discuss what they are doing or ask someone for help (Haugland & Wright, 1997:8). Students' social development can benefit from group work when they are in a position to enquire about things that surprise them while exploring programs, and when they share their results with friends and teachers. The social interactions allow students to learn from several sources, not just the teacher.

Technological tools can also free teachers' time so they can interact with students more. Teachers can leave fact-finding to the computer and spend their time doing what they were meant to do as content experts: arousing curiosity, asking the right questions at the right time and stimulating debate and serious discussion around engaging topics (Hancock, 1997; Morrelli, 1990). Teachers are able to give students more control once they see what students are able to do with technology and how willing and able they are to take responsibility for their own learning (Means & Olson, 1995). While observing students working with computer applications, teachers can see the choices students are making on the monitor or printout, pose questions regarding students' learning goals and decision making and make suggestions for revisions when needed.

Technological tools can generate and manipulate mathematical models. Since modelling problems often arise from real life situations, the availability of technological tools greatly facilitate its feasibility in school mathematics instruction. Often, problems involving modelling are not ready-made. For instance, as Pollak (1986:347) notes, numbers in real world are messy which can be very large or very small. But the burden of lengthy and time-consuming procedures associated with problem-solving can be removed by using technological tools. Technological tools also allow for multiple solutions to realistic problems.

According to NCTM (2003), technological tools can increase both the scope of the mathematical content and the range of the problem situations that are within students' reach. Powerful tools for computation, construction and visual representation offer students access to mathematical content and contexts that would otherwise be too complex for them to explore. Using the tools of technology to work in interesting problem contexts can facilitate students' achievement of a variety of higher-order learning outcomes, such as reflection, reasoning, problem posing, problem-solving and decision-making (NCTM, 2003).

Computers can function as intellectual partners of the learner and assist learners to develop their critical and logical thinking, as well as problem-solving and classification skills. They can also help to teach individual learners what they are ready to learn, at their own pace, through drill-and-practice, discovery and cooperative learning, discussion and reflection (Haugland & Wright, 1997:42).

With computers, mistakes become positive learning experiences because the program immediately helps the students work them through with no one else involved (Roth, 1999:29). Roth notes that with computers, students can "stop the teacher" and go back over a situation as many times as desired, with elaboration, if desired, and not worry about holding the rest of the class up (Roth, 1999:29).

In summary, technology can play a prominent role in today's mathematics instruction. The availability of such tools to school mathematics education might make possible the teaching and learning of mathematics through constructivist methods. It can facilitate group work, interaction, self-regulation (self-observation, self-evaluation and self-reaction), the use of various linked approaches to the same problem situation and the use of the real world problems which are relevant to student experiences.

2.4.2 The Impact of Technology on Mathematics Education

Technology impacts on what is taught and how it is taught, what students learn and how students learn and how the learning is assessed (Beckmann *et al*., 1999:451). Fey (1989:238) asserts that technology influences mathematics education in such a way that it has an impact on the selection of the content and process goals, organisation of teaching and learning environments and assessment of achievement.

In connection with this, Pollak (1986:347) writes:

> The first and most readily apparent effect of technology on the teaching of mathematics is the use of technology in teaching existing mathematics – in helping to overcome the innumerable pedagogic difficulties with which we are so familiar, in helping to motivate students at a place where the background is weak (after the computer has found where that is!), in helping the teacher to do a better job. We can use the microcomputer to provide practice for the student with a new technique, to tutor the student, to show some new applications of the current subject matter, to diagnose a persistent pattern of error, to try out special cases in a situation in which the mathematical pattern is not clear, or to manage a series of individualized tests.

Mathematical Sciences Education Board (in Confrey, 1992:141) suggests that computing devices will decrease on manual skills, increase the importance of topics previously deemed too difficult to teach, emphasize both problem formulation and problem-solving of realistic problems and lead to the creation of tools undreamed of by mathematics educators. In the subsequent sections we will discus the impact of technology on the curriculum (content, educational goals and assessment) and the classroom environment.

a) Content

One influence of technology on mathematics education, particularly the mathematics curriculum, is that it makes certain topics possible to teach - which we have always wanted to include in the curriculum, but which we were simply unable to handle pedagogically (Pollak, 1986:347). A good example of this kind of topics is data analysis. Technology also makes certain topics necessary. For example, discrete mathematics, topics like combinatorics and graphs and logic. These are part of "mathematics for computer science", the tools that have to be available for the student to understand how you do things on a computer and why. Furthermore, because of technology, some new mathematics becomes possible (like fractal geometry) (Waits & Demana, 2000:56) and topics traditionally seen as advanced can be now accessed sooner (like functions) (Arcavi, 1995:157).

The overall priorities in school mathematics have also changed because of technology. What is considered as important for all students simply is not the same as it was (Pollak, 1986:347; Waits & Demana, 2000:55). As Pollak (1986:347) points out, certain topics like estimation are even more important than they used to be. Drill in the elementary operations is less important than before, and it is possible to argue that long division might all but disappear. Less familiar is the thought that much symbolic manipulation is easily done on the microprocessor, so that this aspect of the traditional mathematics (algebra) courses could well be deemphasized.

Technology influences the school mathematics curriculum in such a way that the subject is transformed from a procedurally dominated subject to a study of patterns and relationships (Wheatley & Shumway, 1992:1; Yerushalmy & Gilead, 1997:156). Technology allows for a reorganisation of mathematical concepts within a topic in which more concepts are covered and less emphasis is placed on memorization and manipulative skills (Heid, 1998). Furthermore, it allows students to move quickly and easily beyond the usual computational burden and to experience some of the true richness of the subject (La Torre, 1993:162). Technology also provides students and teachers greater opportunity to engage in more realistic problems so that they can solve problems of real life situations (Fey, 1989; Heid, 1998).

b) Educational Goals

Another impact of technology on mathematics education is that it makes us revise or examine the goals and objectives of our traditional mathematics instruction which has been dominated by mastery of skills and procedures. According to Fey (1989:249), the use of computers has forced to facilitate the reconsideration of curricular objectives in traditional mathematics topics that the potential of computers can be explored. As Fey (1989:266) points out, revision of curricula goals to acknowledge that computers and other electronic information technologies are now standard tools for problem-solving and decision-making will lead to significant changes in what we ask and empower students to learn.

c) Assessment

Technology also impacts on how learning is assessed (Beckmann *et al.,* 1999:451). *Assessment Standards for School Mathematics* (NCTM, 1995) recommends that technology should be an important feature of assessment so that assessment and instruction are aligned. "Not only has technology changed what we teach and how we teach, it has also changed how we test our students" (Laughbaum, 1998:184). By using technological tools in assessment, teachers can more easily measure student growth in conceptual understanding and problem-solving ability through the use of problems that are more open-ended and non-routine (Branca *et al.,* 1992:12).

Teachers can now emphasise the process and not only the product of learning and understanding, not just knowledge (Kinzer, 1986:228). Beckmann *et al.* (1999:451) note that the assessment items that have been used to assess students' understanding of mathematics (functions) are no longer appropriate in a technologically rich classroom environment. Appropriate assessment will naturally follow as school reform incorporates technology into the mathematics curriculum (Matusevich, 1995). Harvey and Osborne (1991:84) state that due to the contribution of the use of technologies to a better conceptual learning, a more extensive knowledge of applications, development of higher order skills and improved problem-solving performance becomes more accessible for assessment by de-emphasising testing of lower level skills.

d) The Mathematics Classroom Environment

Farrell (in Dunham and Dick, 1994:443) notes that students became more active in classrooms in which graphing technology was being used, with more group work, investigation and exploration and real problem-solving. Rich, Simonsen, Beckmann and Davis (in Dunham and Dick, 1994:443) report a shift to fewer lectures by teachers and more investigation by students in technologically rich classrooms.

Harvey and Osborne (1991:75) put teachers' activity as "…a guide on the side instead of a sage on a stage, that is to work with students more closely and on an individual basis instead of lecturing to them, at best, engaging in problem solving with the whole class." Pollak (1986:350) observes that in traditional mathematics instruction, most teaching of mathematics is authoritarian. The teacher expects to act as the fountain of wisdom, to be the boss and behave accordingly. However, in a technologically rich classroom environment, the pedagogy can be changed to a fully participatory pattern in which the teacher acts as moderator of the discussion and not as source of all knowledge.

Heid (1997:24) reports that when using technological tools "…there was less teacher control of the classroom activities and that teachers were less likely to function as authoritative experts and more likely to serve as collaborators." Thus, the use of technologies can change the classroom environment to be more likely interactive in which students get freedom and opportunity to interact with complex mathematical objects and seems to facilitate students' ability to self-regulate (Nicaise & Barnes, 1996:210).

Technology provides students greater opportunities for reflection, discourse and multiple points of views (Nicaise & Barnes, 1996:211). Nicaise and Barnes (1996:208) note that computer technology allows students "to observe and interact with individuals, many of whom have divergent views and opinions. This interaction should provide students with opportunities to react to differing views, challenge others beliefs, and reflect their own ideas." Students who reflect on what they do and communicate with others about it are in the best position to build useful connections (understanding) in mathematics (Hiebert *et al*., 1997:6).

Technology helps students to become actively involved in problem-solving, to talk and read about mathematics and to make generalizations (Waits & Demana, 2000). Instead of learning through direct instruction, a student makes generalizations and constructs knowledge in a way that makes sense to the student. Pollak (1986:351) points out that technology in its best pedagogic use encourages discovery learning. In a technologically rich classroom, students have the opportunity to experiment and find out for themselves. Technology can help to guide each individual student to the "*aha*" of discovery or experience, guided by the inductive knowledge of the student's pattern of thought and individual strengths and instincts (Pollak, 1986:351).

Ruthven (1992:100) points out that during the use of technologies in mathematics instruction "responsibility is devolved to students that play a more active part in developing and evaluating mathematical ideas." Ruthven (1992:100) asserts that students not only grasp ideas but also develop their "capacity to tackle novel situations." That is, students learn mathematical concepts by their active participation in mathematical practice.

In a technologically rich classroom, students are encouraged to do exploration of, and experimentation with mathematical ideas since a more detailed explanation of concepts can be offered due to the relative ease of simulation and drill and practice exercises can be effectively supplemented with realistic problems (Heid, 1998). Nicaise and Barnes (1996:210) note that the use of technologies in mathematics instruction makes students not to depend solely on a teacher but allow them to conjecture, build, test and discover mathematical concepts on their own and evaluate their own ideas. In this kind of learning students' roles in mathematics instruction can undergo a shift from passive receivers of information to becoming more involved in group work, real problem-solving, investigating, symbolising and consulting with technology (Farrell in Dunham & Dick, 1994:443).

The use of technology also enables teachers to provide activities that encourage students to explore mathematical ideas (Norman & Prichard, 1992:260). Once known as the sole disseminators of information, teachers now identify themselves as guides, mentors and facilitators whose roles are to motivate students and engage them in discussion and reflection. As compared to the traditional paper-pencil way of teaching and learning of mathematics, teachers have time to introduce more problem-solving and investigative work and to develop their own teaching styles (Shuard, 1992:33) which best suit their students in particular settings of the classroom environment. Nicaise and Barnes (1996:207) note that the task of the teacher changes from information providers to problem or task presenters or scaffolders. Similarly Dunham (1993:90) points out that teachers continue as managers but less often task setters and explainers and become expert at guiding, questioning, discussing, clarifying and posing mathematical concepts. Cave (1995:372) observes that "In the past, a graph could only be created by hand; therefore, most curricula emphasized the actual graphing of equations. With the help of technology, teachers can now concentrate on teaching students how to investigate what the graphs represent as well as how to interpret the graphs." Fey (1989:251) also says,

> The role of the teacher shifts from demonstration of "how to" produce a graph to explanations and questions of "what the graph is saying" about an algebraic expression or a situation it represents. Students task shift from plotting of points and drawing curves to writing explanations of key graph points or global features.

Shuard (1992:35) reports that the use of technology allowed teachers to develop “a style of talking with students about mathematics that was different from the usual questions-answers evaluation style.” Shuard notes that the teacher no longer pointed students towards the expected ‘correct answers’ to the teacher’s questions but instead asked the students to explain their own thinking and become coordinator of the classroom discussions in such a way that the students are valued and supported but required to do mathematical thinking for themselves (Shuard, 1992:35).

2.4.3 Studies Related to Technology Use

Dunham's review of research reports (in Dunham and Dick, 1994:441-442) indicated that many students who use graphing technology:

- are better able to relate graphs to their equations;
- can better read and interpret graphical information;
- obtain more information from graphs;
- have a greater overall achievement on graphing items;
- are better at "symbolizing," that is, finding an algebraic representation for a graph;
- better understand global features of functions;
- increase their "example base" for functions by examining a greater variety of representations; and
- better understand connections among graphical, numerical and algebraic representations.

The Software Publishers Association (SPA) commissioned an independent meta-analysis of 176 studies focusing on the effectiveness of technology in schools. This report concludes that the use of technology as a learning tool can make a significant difference in, among other things, student achievement as measured by standardized tests (Sivin-Kachula & Bialo, 1996).

Funkhouser (1993) found that high school algebra and geometry students who used computers (problem-solving software) scored significantly higher on mathematics content tests than groups of students who did not use the software. The students using the software also made significant gains in problem-solving ability.

Kulik and Kulik (1991) found that students who were taught using computer technology had higher examination scores than students who were taught by conventional methods without computer technology. Quesada and Maxwell (1994), Alexander (1993), Chandler (1993), Durmus (2000) and Graham and Thomas (2000) also report that students who used this technology obtained significantly higher scores than those students who did not use it.

The Third International Mathematics and Science Study (TIMSS) provides evidence that students who were allowed daily use of calculators performed considerably better on the TIMSS tests than those students who rarely or never used calculators (The International Study Center, 1998). Hollar and Norwood (1999) found that students in graphing approach classes demonstrated better understanding of functions than students in traditionally taught classes. In addition to this, research reports show that by using technology students increased their proficiency in relating functions to their graphical representations and decreased their dependence on memorized rules (Dugdale, 1993) and were able to visualize concepts more easily (Smith & Shotsberger, 1997).

According to Dunham and Dick (1994:443), students who use graphing technology had more flexible approaches to problem-solving, were more willing to engage in problem-solving, worked longer on a problem, concentrated on the mathematics of the problem and not on the algebraic manipulation, solved non-routine problems inaccessible by algebraic techniques and believed calculators improved their ability to solve problems. The use of technology was also found to increase student confidence and interest in mathematics and improve student attitudes (Dunham & Dick, 1994:443).

Although these research results are extremely encouraging, not all results have been positive. Hall (1993), Pankow (1994), Rich (1993), Ritz (1999) and Smith (1996) reported that there were no significant differences in achievement between students who used technology and students who did not use it. Becker (in Dunham and Dick, 1994:442) also found that the use of graphing technology did not improve students' understanding of functions in a college pre-calculus course while Giamati (1991) reported that the use of technology (graphic calculators) affected students' performance negatively. According to Giamati's (1991) report, a control group of students who did not use graphic technology better understood graphical transformations and curve sketching than an experimental group who used graphic technology.

2.5 Conclusion

As discussed above, ample evidence is found in literature and research results that computer technologies can play a significant role in mathematics education. One of the concepts in mathematics where computer technologies may offer a better way of learning than the traditional approach is function. Graphing technologies have the potential to affect teaching and learning in mathematics dramatically, particularly in the fundamental areas of functions and graphs and can empower students to be better problem solvers (Dunham & Dick, 1994:444). The numerical, graphic and symbol manipulation tools provided by computers offer unique kinds of insight and power in mathematical teaching, learning and problem-solving (Fey, 1989:255). Computers can also change the classroom environment into a more interactive environment through constructivist methods. They can facilitate changes in students' and teachers' classroom roles, resulting in a more interactive and exploratory learning environment and can affect students' attitudes positively.

Various researchers have reported that the use of technology improved students' understanding of functions and that students who used technology obtained higher scores than those who did not use it (Alexander, 1993; Chandler, 1993; Durmus, 2000; Funkhouser, 1993; Kulik & Kulik, 1991; Hollar & Norwood, 1999; Graham & Thomas, 2000; The International Study Center, 1998).

Hall (1993), Pankow (1994), Rich (1993), Ritz (1999) and Smith (1996), however, reported that there were no significant differences in achievement between students who used technology and students who did not use it. Yet, they noted that there were some positive effects on other dimensions of students' mathematical learning.

In another study, Becker (in Dunham and Dick, 1994:442) found that the use of graphing technology did not improve students' understanding of functions in a college pre-calculus course, and Giamati (1991) found that students using the traditional paper and pencil method were superior at sketching functions; understanding translations, stretches and shrinks; and describing parameter variations.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

An account of a review of relevant literature was given in chapter 2 and subsequently this chapter will be devoted to a discussion of the design of the empirical investigation. Aspects that will be discussed include methods of research, the research instruments used, the study population and samples of the population.

3.2 Research Design

To investigate the influence of the use of computers in the learning and teaching of mathematics, a review of literature was undertaken and an empirical investigation was done. The empirical investigation comprised quantitative and qualitative research. In conducting the quantitative research, an experimental design and a questionnaire survey were used.

In executing the qualitative research, an interview schedule was used to gather data from selected learners.

3.2.1 Literature Study

The information related to the topic of investigation was gathered from publications, books, journals, dissertations and theses. This helped me to formulate the research problem and gain theoretical knowledge about the influence of the use of computers in the teaching and learning of mathematics.

3.2.2 Empirical Investigation

The present study employed a multi-method approach, which included quantitative and qualitative research to gather relevant data (i.e. to understand in the broadest possible terms the research topic chosen). Quantitative research relies upon measurement and various scales to generate numbers that can be analyzed using descriptive and inferential statistics (Bless and Highson-Smith, 2000:38), and "aims mainly to measure the social world objectively, to test hypotheses and to predict and control human behaviour" (De Vos, 2002:79). By contrast in qualitative research the emphasis is "… on the qualities of entities and on progresses and meanings that are not experimentally examined or measured in terms of quantity, amount, intensity and frequency (Denzin & Lincoln, 2000:8), and "…aims mainly to understand social life and the meaning that people attach to everyday life" (De Vos, 2002:79).

De Vos (2002: 341-342, 365) asserts that combining qualitative and quantitative styles of research and data in a study helps researchers to look at something from several angles so that they can see the different aspects of it. The triangulation technique in social science attempts to map out, or explain in detail, the richness and complexity of human behavior by evaluating different viewpoints with the use of both quantitative and qualitative techniques (De Vos, 2002:341). With reference to the advantages of the multiple method approach, Cohen and Manion (1980:208) write: exclusive reliance on one method may bias or distort the researcher's picture of what is being investigated, therefore the more the methods contrast with each other, the greater the researcher's confidence and the more he can overcome the problem of being bound by methods.

Thus, quantitative and qualitative research methods were selected to investigate what influence the use of computers using MS Excel and RJS Graph software has on grade 11 Eritrean students' understanding of functions in the learning of mathematics. From the qualitative and quantitative nature of the investigation employed, it is believed that the study will allow for some form of generalizations to be made about a wider population after a small selected sample has been studied.

a) Experimental Design

In the present study, a quasi-experimental (a pretest-posttest experimental and control group) design was used to investigate what effect the use of computers has on grade 11 Eritrean students' performances (achievements) in the learning of functions. Two sample groups (i.e. experimental and control groups) of students were involved in the study. The design can be illustrated as follows:

Experimental group:	O_1	X	O_2
Control group :	O_1		O_2

Where:

'X' refers to the independent variable or the treatment given to the experimental group

'O_1' is the first set of observations of the dependent variable (pre-test)

'O_2' is the second set of observations of the dependent variable (post-test)

- **Hypothesis**

To determine whether the use of computers has an effect on students' performances (achievements), the following null hypothesis was formulated.

H_o: *There is no significant difference between the mean scores of students in the experimental and control groups.*

In order to verify the stated null hypothesis a pre-test (task 1) and post-test (task 2) were used. First, the same pre-test (task 1) was given to the two groups before conducting the experiment. This was followed by a teaching course on the concepts of functions, particularly quadratic functions, to both groups for four weeks (approximately 20 hours). The experimental group was taught these concepts with the use of computers using MS Excel and RJS Graph software. The control group was taught the same content in paper and pencil format. Later, after the teaching course (i.e. after the experimental period), the same post-test (task 2) was given to the two groups.

- **Variables**

The *independent variable* in the experimental design is students' exposure to the computers using MS Excel and RJS Graph software while the students' scores on tests is the *dependent variable*. Thus, the experimental design attempts to investigate the cause-effect relationship between the use of computers using MS Excel and RJS Graph software and students' test scores.

b) Questionnaire Survey

A questionnaire survey was also conducted in investigating the influence of the use of computers in the learning of functions. Questionnaire surveys are generally used to investigate people's beliefs, values, attitudes, perceptions, experiences, feelings etc.

A questionnaire has both merits and demerits associated with it. According to Makgato (2003:207), the following are some advantages and disadvantages of a questionnaire.

Advantages	Disadvantages
▪ It saves time ▪ It reduces issues to their basic element ▪ Respondents have more time ▪ Low costs involved	▪ Lacks flexibilities ▪ Inadequate responses or low response rate ▪ No control over the environment

Table 3.1 Advantages and disadvantages of a questionnaire

C) Qualitative Research Aspects

Qualitative research, according to Hitchcock and Hughes (1995:12), enables researchers to learn at first hand about the social world they are investigating. It provides a means of involvement and participation in that world through a focus on what individual actors say or do.

Qualitative research frequently utilizes observations and in-depth interviews. It involves a description in words, exploring to find what is significant in the situation.

Crowl (in Makgato, 2003:32) characterizes qualitative research as follows:

- It takes place in a natural setting and uses the researcher as the key instrument.
- It deals with descriptive data in the form of words and pictures rather than numbers.
- It focuses on process, not merely product.
- It relies on inductive rather than deductive data analysis; and
- It focuses on how different people make sense of their lives.

In the present study, a qualitative approach was applied in conducting interviews with selected learners.

3.3 Population

The population of this study is Eritrean grade 11 students. The study was conducted in Red Sea (Keih Bahri) Secondary School in Asmara, the capital city of Eritrea. The reasons for the selection of this school as a site of study were:

i) The school has a relatively big computer lab and continuous electricity supply.

ii) The school has a relatively good transport system.

3.4 Process of Data Collection

3.4.1 Research Instruments

Data for this study were collected using a pre-test (task 1) and a post-test (task 2), a questionnaire and an interview schedule.

- ***Pre-test (task 1)***

A pre-test (task 1) was designed to investigate the equivalence of the experimental and control groups. This was administered to the students in both the experimental and control group prior to the experiment. If the means of the performances of the two groups do not differ significantly, it can be assumed that the two groups are comparable. The focus questions in the task were functions (quadratic functions). (*See appendix A*).

Students from each group were given 60 minutes to complete the pre-test. The scheme of evaluation (scoring) of marks for each question in the pre-test was as follows:

Mark or score in percentage per problem	Observed characteristics of the student's work (solution)
0%	▪ No attempt (blank paper) ▪ Numbers from problem recopied – no understanding of problem evidenced ▪ Incorrect answer and no work shown
20%	▪ Inappropriate strategy started - problem not finished ▪ Approach unsuccessful - different approach not tried ▪ Attempt failed to reach a sub-goal
40%	▪ Inappropriate strategy - but showed some understanding of the problem ▪ Appropriate strategy used - did not find the solution; or reached a sub-goal but did not finish the problem ▪ Correct answer and no work shown
60%	▪ Appropriate strategy but ⇒ ignored a condition in the problem ⇒ incorrect answer for no apparent reason ⇒ thinking process unclear
80%	▪ Appropriate strategy or strategies ▪ Work reflects understanding of the problem ▪ Incorrect answer due to a copying or computational error
100%	▪ Work shown clearly and correct answer (appropriate solution process or processes and correct answer)

Table 3.2 Scheme of evaluation (scoring) for the questions in the pre-test (task 1) and post-test (task 2)

- ***Post-test (Task 2)***

A post-test (task 2) was designed and administered at the end of the experiment to students in both the experimental and control groups. If the mean performance of the experimental group is significantly different from the mean performance of the control group, it can be assumed that the performance of learners must have been influenced by the use of computers using MS Excel and RJS Graph software. The questions in the task focused on functions (quadratic functions). (*See appendix B*).

Students from each group were given 150 minutes to complete the post-test (task 2). The same evaluation scheme that was used for the pre-test (task 1) was used to evaluate each question in the post-test (task 2).

- ***Questionnaire***

A ten-item questionnaire, consisting of closed questions, was designed and administered only to the experimental group as a whole after the experimental period. Respondents were required to indicate their choices (the extent to which they agree or disagree) to each item on a Likert type scale: strongly disagree, disagree, agree and strongly agree. The data obtained from the respondents were analyzed statistically. (*See appendix C for the questionnaire*).

The purpose of the questionnaire was to investigate the influence of the use of computers on students' motivation, attitude, problem-solving (engagement with and exploration of mathematical ideas), group work and cooperation and discussion among students and between students and the teacher in the learning of mathematics (functions).

- ***Interview Schedule***

An interview schedule is an instrument that can be used to gather in-depth information from an individual. It is used to obtain in-depth information about a participant's thoughts, knowledge, reasoning, motivations, attitudes, perceptions, experiences and feelings about a topic (Johnson & Christensen, 2000:144).

An interview has the following advantages (Bailey, 1994:174; Sax, 1979:23).

- The interview is flexible and applicable to many different types of problems. It is flexible in the sense that the interviewer may change the mode of questioning if the occasion demands. If the responses given by the subject are unclear, questions can be rephrased.
- It is useful in collecting personal information, experiences, attitudes, perceptions or beliefs by probing for additional information.
- It promotes motivation and openness. Almost all interviews attempt to develop rapport between the interviewer and the respondent (interviewee). Once interviewees accept the interview as a non-threatening situation, they are more likely to be open and frank. This openness adds to the validity of the interview.

The following are some disadvantages of an interview.

- It is time-consuming to transcribe.
- It produces a large amount of redundant texts.
- It is expensive to use and often difficult to administer.
- It may introduce elements of subjectivity and bias, and rapport may cause the interviewee to respond in a certain way to please the interviewer.

An interview can be structured, semi-structured or unstructured.

The s*tructured interview* can be described as a closed situation where the interviewer has little freedom to make modification, the content and procedures are organized in advance, and the sequence and wording of the questions are determined by means of a schedule. The respondents' answers are not followed up to obtain greater depth (Gall, Borg & Gall, 1996: 310).

The *semi-structured interview* involves a number of pre-determined questions or several topics, which are typically asked in a systematic and consistent order, but it does allow freedom for modification to some degree. A semi-structured interview permits the researcher to probe far beyond the answer to his/her prepared standardized questions to obtain more information. The interviewer asks reasons or explanations for the interviewees' answers.

The *unstructured interview* can be described as an open situation, having greater flexibility and freedom. It does not involve a detailed interview guide. Instead, the interviewer asks questions that gradually lead the respondent to give the desired information (Gall *et al.,* 1996: 310-311).

In the present study, a *semi-structured* interview schedule, consisting of five open-ended questions, was designed and conducted after completion of the experiment. The participants in the interview were four students from the experimental group, selected using purposeful sampling. The four students were selected for interviews because of their computer skills and regular attendance during the experimental period. The interviews, which were conducted in the students' mother tongue, were recorded using a video recorder and later transcribed. Each interview lasted about 10 - 15 minutes. The interviews were conducted by myself. (*See appendix D for the interview schedule*).

The purpose of the interview was to investigate the influence of the use of computers on students' motivation, attitude, problem-solving (engagement with and exploration of mathematical ideas) and the classroom environment (students' group work and participation, cooperation and discussion among the students and discussion between the students and the teacher) in the learning of mathematics (functions in particular).

3.4.2 Sampling

The school in which the experiment was conducted has a morning shift and an afternoon shift. Students who were attending classes in the morning shift were free from attending classes in the afternoon shift and students who were attending classes in the afternoon shift were free from attending classes in the morning shift throughout the school year.

For the purpose of the study, 15 students from each of the grade 11 morning and afternoon shifts of the school were randomly selected. This gives a total of 30 students in the study. A table of random numbers was used in this regard. Thereafter, the two groups of students (the one from the morning shift and the other from the afternoon shift) were assigned randomly to a control group and an experimental group. The experimental teaching for both groups was done during the free shift of the school time.

3.4.3 Process

In order to conduct the study, I discussed the process with the principal, vice principal and mathematics teachers of the school. All of them co-operated very well with me, especially in the arrangements for the time needed for the lessons and the selection process for participating students.

After selecting the students, both the experimental and control groups were given the same pre-test (task 1). This was followed by teaching the concepts of functions, particularly quadratic functions, to both groups. I was the teacher for both groups.

The teaching for both groups lasted for four weeks (approximately 20 hours). During the four-week project, the activities and contents were the same for both groups. Each student in the experimental group was provided with a personal computer while each student in the control group was provided with an exercise book, a pencil, a pen, a maths instrument box and an eraser. Students in both groups were also provided with mathematics learning materials on the topics “introduction to relations and functions” and “quadratic functions”.

Each student in the experimental group received an orientation on how to use the commands (menus and buttons of the toolbars) of the computer software (MS Excel and RJS Graph software). The control group learned the concepts of quadratic functions through the traditional method. That is, they were not provided with any computing and graphing technology.

After the experimental period both groups were given the same post-test (task 2). The questionnaires were administered to each learner of the experimental group while four students of the experimental group were interviewed.

Before administering the research instruments, I requested experienced secondary school mathematics teachers and lecturers to comment on the suitability of these instruments. They gave comments and suggestions for improvements which were made.

I also conducted a pilot testing for the interview schedule with two students from the experimental group after completion of the experiment. The pilot testing was very helpful to test the interview schedule and improve it before it was administered.

3.5 Statistical Procedures and Techniques

The data collected using a pre-test (task 1) and a post-test (task 2) were analysed using a *t-test* for independent groups to determine whether there was a significant difference between the mean scores of the experimental and the control groups. A Student’s t-test for independent sample groups is used.

$$t = \frac{M_1 - \mathrm{M}_2}{\sqrt{\frac{(N_1 - 1)S_1^{\,2} + (N_2 - 1)S_2^{\,2}}{N_1 + N_2 - 2}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

Where:

M_1 = mean of the experimental sample group

M_2 = mean of the control sample group

N_1 = numbers of cases in the experimental sample group

N_2 = numbers of cases in the control sample group

S_1^2 = variance of the experimental sample group

S_2^2 = variance of the control sample group

3.6 Reliability

In this study, the reliability and validity of the instruments (and data collected) were considered. The description of quality instruments used to collect data typically deals with these two related concepts, reliability and validity.

Reliability means consistency of the research instruments used to measure particular variables. Obtaining the same results when the instruments are administered again in a stable condition guarantees reliable instruments (De Vos 2002:168; Mlangeni in Makgato, 2003:210). According to Schuyler (in Makgato, 2003:210), researchers evaluate the reliability of instruments from different perspectives, but the basic question that cuts across various perspectives (and techniques) is always the same: To what extent can we say that the data are reliable? To ascertain how reliable are the measuring instruments that were used in this study, reliability coefficients (Cronbach's alpha) were calculated.

3.7 Validity

According to Ary, Jacobs and Razavieh (1990:250), the term validity refers to the extent to which an instrument measures what it intends to measure. Validity addresses the following two questions (De Vos, 2002:166):

- What does the research instrument measure?
- What do the results mean?

The core essence of validity is captured nicely by the word *accuracy*. From this general perspective, a researcher's data are valid to the extent that results of the measurement process are accurate.

The following process was implemented to ensure the validity of the research instruments:

Before the pre-test (task 1) and post-test (task 2) were administered to students, they were verified and examined by lecturers and experienced secondary school mathematics teachers. Two lecturers and three experienced secondary school mathematics teachers were consulted on the wording and nature (content) of the items (questions) in the pre-test (task 1) and post-test (task 2) and the time required to answer the questions. Their opinions on content, semantics, relevancy and time were sought. I also sent the post-test (task 2) to professor DCJ Wessels (my initial supervisor). He gave me the necessary comments and suggestions. Moreover, the students are accustomed to testing of a similar nature in the school. Regarding the questionnaire and interview, I sent the items of the questionnaire and the questions of the interview schedule to professor DCJ Wessels, before administering them. He gave me the necessary comments. I also consulted with two lecturers. In addition, I conducted a pilot testing for the interview schedule with two students from the experimental group after completion of the experiment in order to identify possible mistakes or problems or deficiencies and correct them before the instrument is administered. Pilot testing is very helpful as it makes a researcher aware of any possible unforeseen problems that may emerge during the main investigation (Ntsohi, 2005:11). Based on the opinions and comments I got from the teachers and lecturers and the pilot testing, the instruments were amended. Therefore, after the wide consulting of experts, incorporating their opinions and comments as well as pilot testing, it may be concluded that the instruments portray the desired level of content validity.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.1 Introduction

In the preceding chapter, the research design and methodology that was followed in conducting the study was discussed. A pre-test (task 1) and a post-test (task 2), a questionnaire and an interview schedule were used. In this chapter results of the investigation will be presented, analysed and interpreted.

4.2 Pre-test (Task 1)

The students' solutions to questions posed in the pre-test as well as the post-test were marked in terms of the scheme of evaluation of full, partial and no mark as described in section 3.4.1. From this, the scores of students and descriptive statistics were calculated and a null hypothesis was tested. This was followed by a reliability analysis.

Table 4.1 presents the scores of students in the experimental and control groups for the pre-test (task 1).

Experimental group		Control group	
Students	Marks (Out of 50)	Students	Marks (Out of 50)
S1	27.0	S1	18.5
S2	34.0	S2	22.0
S3	26.0	S3	18.0
S4	24.0	S4	35.0
S5	13.5	S5	23.5
S6	18.5	S6	25.0
S7	30.0	S7	29.5
S8	13.5	S8	15.5
S9	29.5	S9	17.0
S10	31.0	S10	20.0
S11	30.0	S11	26.5
S12	11.5	S12	23.0
S13	34.0	S13	34.0
S14	31.5	S14	18.5
S15	16.0	S15	16.0

Table 4.1 Scores (marks) of students in the experimental and control groups for the pre-test

Statistics	Experimental Group	Control Group
Minimum	11.5	15.5
Maximum	34.0	35.0
Range	22.5	19.5
Mean	24.6667	22.80
Standard deviation	7.95	6.21

Table 4.2 Descriptive statistics of the experimental and control groups for the pre-test

- **Hypothesis testing**

The following null hypothesis was tested in terms of the results of the pre-test as well as the post-test:

H_o: *There is no significant difference between the mean scores of students in the experimental and control groups.*

In both cases the null hypothesis was tested at the 0.05 level of significance. That is, the null hypothesis was rejected if $t_{calculated} \geq t_{critical}$ and accepted if $t_{calculated} < t_{critical}$. Student's t-test for independent groups (see section 3.5) was used to compare the two mean scores of the groups. The Student's t-test was used because the samples were small. Best (1977:283) points out that when small samples (fewer than 30 observations in number) are involved, the Student t-test proves to be an appropriate test to determine the significance of the difference between the means of two independent groups. Table 4.3 shows the results of the t-test application on the pre-test scores.

Group	Mean	Standard deviation	Calculated t	Critical t at 0.05 level of significance and 28 degrees of freedom
Experimental	24.6667	7.95	0.7167	2.048
Control	22.80	6.21		

Table 4.3 Results of Student's t-test application on the pre-test scores

The results in table 4.3 show that the null hypothesis cannot be rejected (calculated t-value is less than critical t-value at the 0.05 level of significance). This means that there is no significant difference between the mean scores of the experimental and control groups of students for the pre-test. Hence, we can conclude that the two groups of students were comparable at the pre-test stage (i.e. before the experiment).

- **Reliability analysis**

Using the SPSS statistical software, reliability coefficients (Cronbach alphas) were calculated to determine the reliability of the instruments (the pre-test and post-test). A reliability coefficient of 0.7 or higher is a desired reliability coefficient that can lead us to say that the instrument (test) is reliable. The Cronbach alpha values that were calculated for the pre-test are indicated in table 4.4.

Item	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Alpha If Item Deleted
1a	17.8333	9.7989	0.8756	0.8541
1b	17.9000	10.2310	0.7399	0.8645
2i	17.9000	9.6103	0.8334	0.8558
2ii	17.9333	9.5816	0.8561	0.8540
3a	18.3000	11.7345	0.3795	0.8873
3b	18.4333	11.8402	0.3691	0.8877
3c	18.2000	11.8897	0.4064	0.8862
3d	18.4667	10.8782	0.4794	0.8835
3e	18.8667	10.1885	0.7445	0.8641
4	18.8667	9.5678	0.5580	0.8877
N of Cases = 30.0		N of Items = 10	Overall alpha = 0.8846	

Table 4.4 Reliability analysis for the pre-test: Item-total statistics and Cronbach alpha values

The alpha value for the pre-test as a whole is 0.8846 which is quite an acceptable reliability coefficient. Therefore, the test can be considered as a reliable instrument to measure students' performances.

Besides the satisfactory item and item-total reliability coefficients (alpha values), all the items show acceptable correlations with the item-total. The lowest value (r) is 0.3691 which implies that the correlation of even this item (item 3b) with the rest of the items (item-total) is acceptable. This correlation point to a 14% (r^2=0.136) overlap between the item and the basket of items which is an indication that even this item shows sufficient homogeneity with the other items. On the other hand, the range of item-total correlation coefficients shows that there is sufficient diversity among the items. These observations, that the test items show adequate homogeneity but also sufficient diversity, is an indication that the instrument (pre-test) portray the required construct validity.

4.3 Post-test (Task 2)

Table 4.5 presents the scores of students in the experimental and control groups for the post-test (task 2).

Experimental group		Control group	
Students	Marks (Out of 70)	Students	Marks (Out of 70)
S1	50	S1	40
S2	66	S2	41
S3	59	S3	46
S4	61	S4	60
S5	38	S5	37
S6	61	S6	43
S7	Drop out	S7	48
S8	53	S8	Drop out
S9	60	S9	36
S10	58	S10	58
S11	60	S11	57
S12	40	S12	43
S13	62	S13	59
S14	65	S14	33
S15	66	S15	32

Table 4.5 Scores (marks) of students in the experimental and control groups for the post-test

Statistics	Experimental Group	Control Group
Minimum	38.0	32.0
Maximum	66.0	60.0
Range	28.0	28.0
Mean	57.0714	45.2143
Standard deviation	8.8619	9.8072

Table 4.6 Descriptive statistics of the experimental and control groups for the post-test

- **Hypothesis testing**

The same null hypothesis that was tested in the pre-test was tested in the post-test. The same statistical test (see section 3.5) that was used for the pre-test was also used to test the null hypothesis for the post-test. Table 4.6 shows the results of the t-test application on the post-test scores.

Group	Mean	Standard deviation	Calculated t	Critical t at 0.05 level of significance and 26 degrees of freedom
Experimental	57.0714	8.8619	3.3564	2.056
Control	45.2143	9.8072		

Table 4.7 Results of Student's t-test application on the post-test scores

The results in table 4.6 show that the null hypothesis can be rejected (calculated t-value is greater than critical t-value at the 0.05 level of significance). This means that there is significant difference between the mean score of the experimental and the control groups of students for the post-test. Hence, we can conclude that the use of computers had a significant effect on students' performances (achievements).

- **Reliability analysis**

To determine reliability of the instrument (post-test), Cronbach alpha coefficients were calculated using the SPSS statistical software and the results are indicated in table 4.8.

Item	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Alpha If Item Deleted
1a	46.0357	56.2579	0.8755	0.9297
1b	45.7143	62.8929	0.0000	0.9389
1c	45.6786	62.7302	0.0000	0.9389
2a	46.1429	53.4603	0.8714	0.9281
2b	46.0000	56.3704	0.7568	0.9311
2c	46.6429	56.3704	0.7568	0.9311
2d	46.7857	53.5079	0.7702	0.9304
3[a]	46.2500	57.3056	0.6719	0.9326
3b	46.5000	60.4815	0.3191	0.9376
3c	46.2143	60.3122	0.3043	0.9380
3d	46.7143	57.1746	0.5634	0.9344
3e	46.6071	55.8029	0.6952	0.9319
4	46.0714	53.9206	0.7159	0.9317
5a	46.0000	57.1852	0.7665	0.9315
5b	45.9286	57.1058	0.7133	0.9320
5c	46.3571	53.3492	0.8284	0.9289
6a	46.1786	56.7447	0.7485	0.9314
6b	46.1786	57.0410	0.4566	0.9375
6c	46.8571	53.6085	0.6764	0.9332
7	46.8929	54.6177	0.7104	0.9316
N of Cases = 28.0		N of Items = 20	Overall alpha = 0.9363	

Table 4.8 Reliability analysis for the post-test: Item-total statistics and Cronbach alpha values

The overall alpha value for the post-test is 0.9363 which is an excellent reliability coefficient. Therefore, the test can be considered as a reliable instrument to measure students' performances.

Besides the satisfactory item and item-total reliability coefficients (alpha values), all the items, except items 1b and 1c, show acceptable correlations with the item-total. As indicated in table 4.8, the item-total correlations of items 1b and 1c are 0 which means that these items (items 1b and 1c) are not correlated with the item-total. But the item-total correlations of the other items are greater than 0.3 which means that these items are adequately correlated with the item-total. That is, each item of the test (post-test), except items 1b and 1c, show sufficient homogeneity with the other items. Furthermore, the range of item-total correlation coefficients shows that there is sufficient diversity among the items. Hence, we can conclude the instrument (post-test) portray the required construct validity.

4.4 Questionnaire

The students responded to items in the questionnaire on the following four point Likert type scale: strongly disagree, disagree, agree and strongly agree. From this, the frequencies and percentage distribution of frequencies of students' responses to items were calculated and this was followed by a reliability analysis.

Figures 4.1and 4.2 and tables 4.9 and 4.10 show the responses of students in the experimental group on the questionnaire.

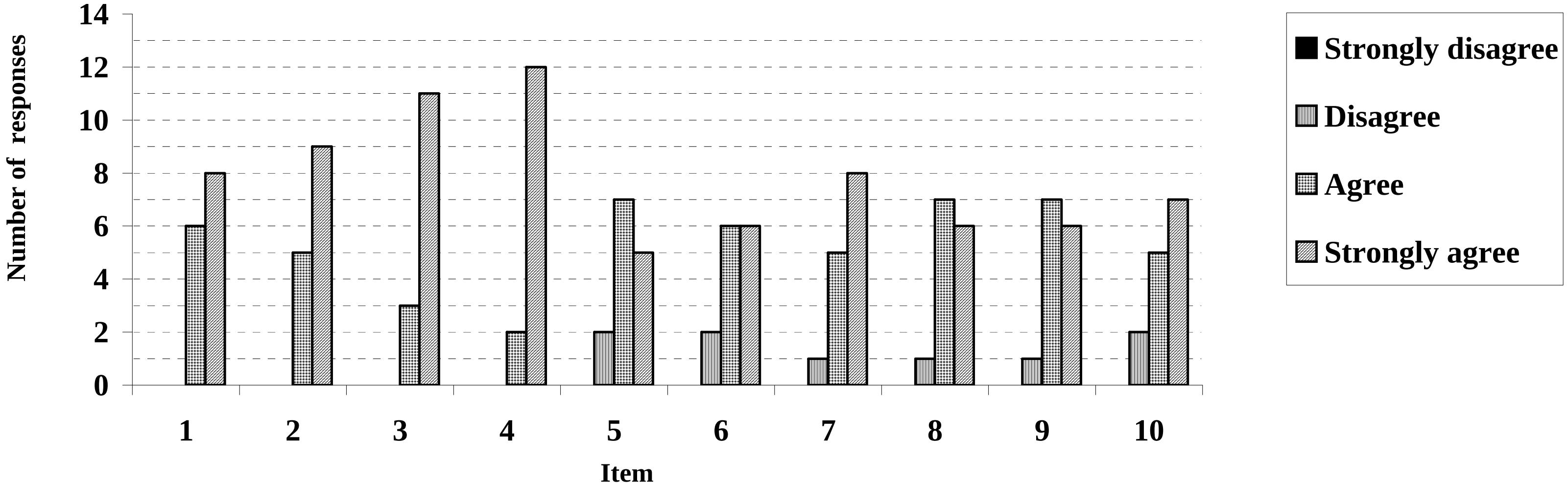


Figure 4.1 A bar chart of students' responses on questionnaire items

Response	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8	Item 9	Item 10
Strongly disagree	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Disagree	0%	0%	0%	0%	14%	14%	7%	7%	7%	14%
Agree	43%	36%	21%	14%	50%	43%	36%	50%	50%	36%
Strongly agree	57%	64%	79%	86%	36%	43%	57%	43%	43%	50%

Table 4. 9 A percentage frequency table of students’ responses on questionnaire items

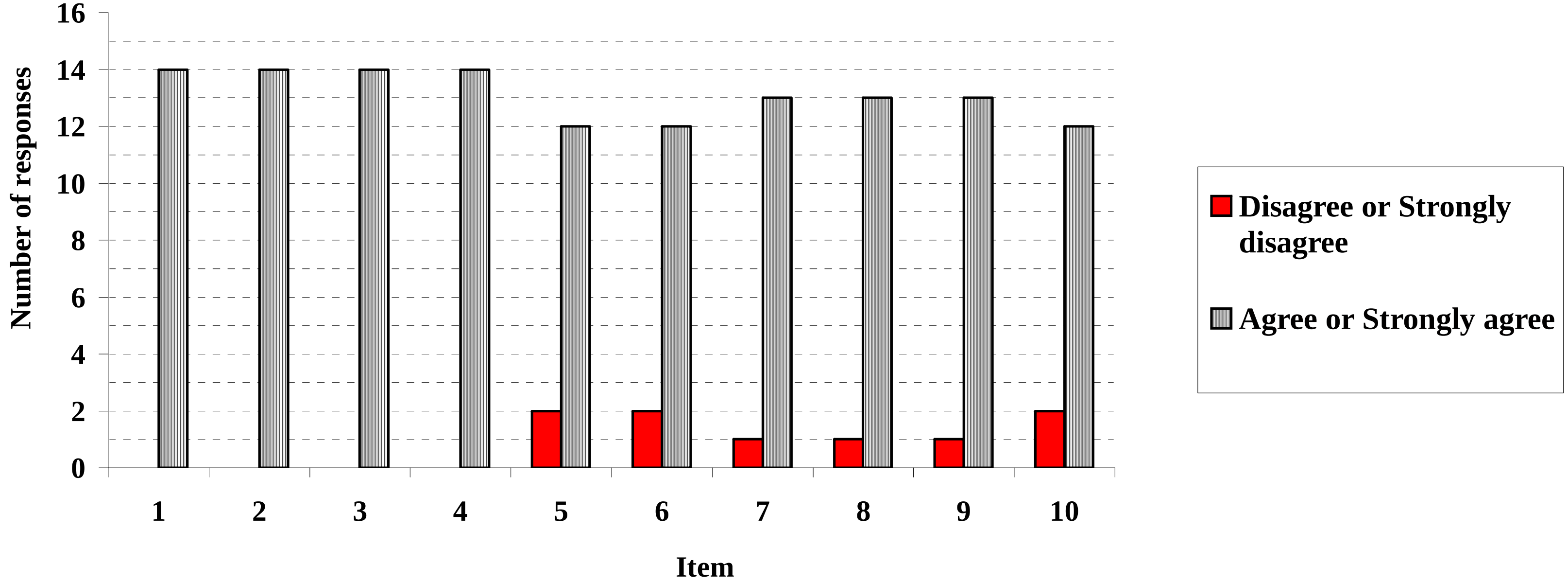


Figure 4.2 A bar chart of students' agree/disagree responses on questionnaire items

Response	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8	Item 9	Item 10
Strongly disagree or Disagree	0%	0%	0%	0%	14%	14%	7%	7%	7%	14%
Agree or Strongly agree	100%	100%	100%	100%	86%	86%	93%	93%	93%	86%

Table 4. 10 A percentage frequency table of students' agree/disagree responses on questionnaire items

The responses to items 1 and 2 as presented in figure 4.2 and table 4.10 above indicate that the use of computers can encourage students to learn quadratic functions and produce a positive disposition in students towards quadratic functions. All the respondents agreed that quadratic functions is an interesting topic to learn using a computer (MS Excel and RJS Graph software) and that it is convenient to solve a quadratic function problem using a graph if you use these software.

The responses to items 3, 4, 5, 9 and 10 as presented in figure 4.2 and table 4.10 above indicate that the use of computers can facilitate students to engage with and explore the nature and properties of quadratic functions and their graphs easily and quickly, work on their own problems and solve real life mathematics problems. All respondents agreed that the use of a computer using MS Excel and RJS Graph software in learning quadratic functions enables one to create tables of values of the functions quickly and draw their graphs easily. Of the respondents, 86% agreed that the use of a computer using MS Excel and RJS Graph software in learning quadratic functions enables one to get sufficient time to investigate the nature and properties of quadratic functions and their graphs. Respectively 93% and 86% of the respondents also agreed that the use of computers in learning quadratic functions gives a student the opportunity to work on his/her own problems and engage with real life mathematics problems.

The responses to items 6, 7 and 8 as presented in figure 4.2 and table 4.10 above indicate that the use of computers can facilitate students' group work and discussion (interaction) among themselves and between students and the teacher. Of the respondents, 93% agreed that the use of computers in learning quadratic functions gives students the opportunity to interact among themselves and with their teacher. Of the respondents, 86% also agreed that the use of computers in learning quadratic functions gives students the opportunity to work in a group.

- **Reliability Analysis**

Using the SPSS statistical software, reliability coefficients (Cronbach alpha) were calculated to determine the reliability of the questionnaire as a measuring instrument. A value of 0.7 or higher is an acceptable value that can lead us to say that the questionnaire is a reliable instrument. The Cronbach alpha values that were obtained are indicated in table 4.11.

Item	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Alpha If Item Deleted
1	31.3571	17.1703	0.8004	0.9050
2	31.2857	18.3736	0.5207	0.9184
3	31.1429	17.8242	0.7885	0.9079
4	31.0714	18.6868	0.6440	0.9150
5	31.7143	16.3736	0.7029	0.9098
6	31.6429	16.0934	0.7242	0.9088
7	31.4286	16.4176	0.7589	0.9059
8	31.5714	17.0330	0.6517	0.9123
9	31.5714	16.7253	0.7170	0.9084
10	31.5714	15.6484	0.7868	0.9046
N of Cases = 14.0		N of Items = 10	Overall alpha = 0.9181	

Table 4.11 Reliability analysis for the questionnaire: Item-total statistics and Cronbach alpha values

The overall alpha coefficient for the questionnaire is 0.9181 which is an excellent reliability coefficient. This value indicates that the questionnaire can be considered as a reliable measuring instrument. Furthermore, the item-total correlations of all the items, as indicated in table 4.11, are greater than 0.5 which means that each item of the questionnaire is adequately correlated with the rest of the items (item-total). On the other hand, the range of item-total correlation coefficients shows that there is sufficient diversity among the items. These observations, that the questionnaire items show adequate homogeneity but also sufficient diversity, is an indication that the instrument (questionnaire) portray the required construct validity.

4.5 Interview

4.5.1 Question 1

Question 1 dealt with students' view on the status quo of the teaching and learning of school mathematics. Respondents were asked to comment on the way in which they are being taught. All four students gave similar responses. Some of the responses to the question: "How are you being taught mathematics?" were:

S1: *First, our teacher tells us formulas and shows us some examples. After that he gives us exercises (class and home work) and corrections on our exercises. Lastly, he (teacher) asks us if we have questions.*

S4: *First, our teacher tells us the rules and procedures that we have to follow to solve a problem using examples. After that he gives us exercises and corrections on our exercises.*

The responses to question 1 indicate that the students were passive in the teaching and learning process. It seems that there is no sign of a problem-centered approach of any kind and that problem-solving activities or exploration of mathematical ideas by students as well as classroom social interaction are lacking.

4.5.2 Question 2

Question 2 dealt with students' experiences of learning quadratic functions using paper and pencil. During the interview, the respondents pointed out the difficulties they were facing in learning quadratic functions. Some of the responses were:

S1: *We were spending a lot of time in computing and plotting points.*

S2*: Because of the repeated algorithmic computations, drawing graphs using paper and pencil was boring… When we make mistakes, it is boring to do it on another sheet of paper again.*

S3 and **S4** gave similar responses.

The responses to question 2 indicate that for the students the construction of graphs of quadratic functions using paper and pencil was time consuming and boring. This may affect the students' attitudes towards mathematics in general, and towards quadratic functions in particular, negatively.

4.5.3 Question 3

Question 3 dealt with the helpfulness of computers (MS Excel and RJS Graph software) in learning quadratic functions. Some of the responses were:

S1*: Previously, when we were learning mathematics using paper and pencil, most of the time, I was dependent on the teacher. I mean that I was waiting until our teacher gives us exercises and corrections for them. But now (during the experiment), I worked on many other quadratic function problems in addition to the exercises which were given by the teacher. This was because the computer was helpful in creating tables of values quickly and drawing graphs easily when I was working on the problems. The computer removed the difficulties that we were facing in drawing graphs.*

S2: *For me, drawing graphs using paper and pencil was boring. I was spending a great deal of time in creating tables of values and plotting points. Especially finding the values of y (dependent variable) by assigning big or fraction numbers for x (independent variable) was a terrible work. I also couldn't plot the points exactly. But now (during the experiment), I was able to create the table of values quickly and draw graphs easily. The computer was very helpful in this regard.*

Furthermore, **S2**, said that

> *I don't feel good to ask the teacher repeatedly when I don't understand something or when I make a mistake. But, with a computer, I can repeat as many times as desired and go back and forth until I grasp the concept.*

S3: *I was able to draw graphs of many quadratic functions within a short time using the computer. This helped me to explore the properties of the functions from its graph quickly and easily.*

S4: *Constructing graphs of quadratic functions using paper and pencil was boring. Previously (using paper and pencil) when I draw the graph incorrectly, I was forced to draw it on another sheet of paper, which was tiresome. But now (during the experiment), I could automatically retry and check the result on the computer.*

The responses to question 3 indicate that the use of computers can ease some of the difficulties that students are facing in learning quadratic functions and improve their understanding of quadratic functions. That is, the use of computers in the learning of quadratic functions can give students the opportunity to engage with and explore the nature and properties of quadratic functions and their graphs actively and facilitate students to develop self-regulation (self-observation, self-evaluation and self-reaction).

However, as one of the respondents pointed out, the use of MS Excel and RJS Graph software has some constraints in learning quadratic functions. The respondent mentioned that:

S4: *Unlike teaching using a blackboard, the software used (MS Excel and RJS software) do not show all the processes (algorithmic computations and plotting points) involved in drawing graphs.*

4.5.4 Question 4

Question 4 dealt with students' experience of learning quadratic functions using computers. More specific, the question aimed at determining what influence the use of computers had on non-cognitive dimensions of students such as motivation and attitude as well as the relationships they had with other students and with their teacher during the experiment. The responses regarding their attitudes and motivation towards quadratic functions were:

S1: *I enjoyed learning* quadratic *functions using a computer… I was working with full concentration. Everybody was also busy and doing something with the computer.*

S2: *Learning using a computer uplifted my interest to learn mathematics and encouraged me to work on many quadratic function problems.*

S3: *Learning quadratic functions using a computer was good…I was encouraged to learn more about quadratic functions.*

S4: *Same as S2.*

The responses of the four students regarding the relationship they had with the other students during the experiment were as follows:

- *We were comparing and discussing the solutions of the problems that we worked on.*
- *We were working individually and in a-group.*
- *We were sharing views and giving comments to each other.*

The responses regarding the relationship they had with the teacher during the experiment were as follows:

S1: *The relationship was simple and friendly.*

S4: *It (relationship) was good. I was asking help and getting assistance from the teacher when I was confronted with problems.*

S2 and **S3** responded similarly.

The responses to question 4 indicate that the use of computers can encourage and motivate students to learn quadratic functions, produce positive attitudes in students towards quadratic functions in particular and towards mathematics in general and facilitate students' group work and participation, cooperation and discussion among themselves and between the students and the teacher.

4.5.5 Question 5

In question 5 students were given the opportunity to convey, if any, additional views or suggestions. The four students suggested the following.

- *Computers are helpful to overcome the difficulties that we were facing in learning mathematics. So, they should be introduced in mathematics classrooms.*

- *Computers are helpful in solving mathematical problems. So, they should be incorporated as learning tools in mathematics classrooms*.

The responses to question 5 indicate that students want computers to be available in mathematics classrooms as learning tools. The availability of these tools in mathematics classrooms may encourage students to participate actively and explore mathematics on their own.

4.6 Conclusion

The aim of the empirical investigation was to determine what influence the use of computers using MS Excel and RJS Graph software has on students' understanding of functions in the learning of mathematics. The findings from the pre-test (task 1) show that the two groups were comparable at the pre-test stage (i.e. before the experiment).

The analysis of the post-test (task 2) results indicates that the use of computers influenced students' performance (achievement) positively. In the post-test (task 2), the mean performance of the experimental group was significantly higher than the mean performance of the control group.

The questionnaire was administered to the entire experimental group. The analysis of the questionnaire data indicates that the use of computers can positively influence students' understanding of functions in terms of problem-solving (engagement with and exploration of mathematical ideas), their motivation to be involved and attitude towards functions as well as facilitating group work and discussion (interaction) among themselves and between students and the teacher.

The responses to the questions in the interview confirmed the results obtained using the other measurements in the sense that the use of computers can positively influence students' understanding of functions in terms of problem-solving (engagement with and exploration of mathematical ideas), motivation, attitude and the classroom environment. That is, the use of computers can ease some of the difficulties that students are facing in learning quadratic functions and facilitate students to engage with and explore the nature and properties of quadratic functions and their graphs, observe and evaluate their work. It can also encourage and motivate students to learn quadratic functions, produce positive attitudes in students towards quadratic functions in particular and mathematics in general, and can facilitate students' group work, participation, cooperation and discussion among the students and between the students and the teacher.

However, the use of MS Excel and RJS Graph software can have some constraints in learning quadratic functions. As one of the respondents pointed out during the interview, the software used (MS Excel and RJS Graph software) do not show all the processes (algorithmic computations and plotting points) involved in drawing graphs in the same way as teaching using a blackboard.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The purpose of this study was to investigate whether the use of computers in the teaching and learning of mathematics (functions in particular) influences students' understanding of functions as reflected in their achievement, motivation, attitude, problem-solving skills (engagement with and exploration of mathematical ideas), group work and cooperation and discussion among students and between students and the teacher. This chapter summarizes the findings, draws conclusions and makes recommendations.

5.2 Summary of the Findings

5.2.1 Summary of the Literature Review

Most of the findings reported in the literature indicated that technological tools such as computers and calculators have a positive impact on students' performances (achievements). Alexander (1993), Chandler (1993), Durmus (2000), Funkhouser (1993), Hollar and Norwood (1999), Kulik and Kulik (1991), Graham and Thomas (2000), Quesada and Maxwell (1994) and The International Study Center (1998) reported that students who used technology obtained higher scores than those students who did not use it. In addition to this, research reports show that by using technology students were able to visualize concepts more easily (Smith & Shotsberger, 1997), increased their performance on standardized test (Sivin-Kachela & Bialo, 1996), increased their understanding of mathematical concepts and decreased their dependence on memorised rules (Dugdale, 1993).

Dunham's review of research reports (in Dunham and Dick, 1994:443) also indicated that students who use graphing technology:

- are better able to relate graphs to their equations;
- can better read and interpret graphical information;

- obtain more information from graphs;
- have a greater overall achievement on graphing items;
- are better at "symbolizing," that is, finding an algebraic representation for a graph;
- better understand global features of functions;
- increase their "example base" for functions by examining a greater variety of representations; and
- better understand connections among graphical, numerical and algebraic representations.

Computer technology allows students to graph functions more easily, quickly and accurately; to manipulate the graphs; and to develop generalizations about the functions. It offers students the opportunity to explore the concepts and notion of functions in multi-representational (symbolic, numeric, tabular and graphic or visual) modes (Beckmann *et al.,* 1999:451; Confrey, 1992:150; Fey, 1989:255; Heid, 1998) and help students to make connections between mathematical ideas (Smith & Shotsberger, 1997), between a real world phenomenon and its mathematical representations and between a student's everyday world and his/her mathematical world (Heid, 1998).

Computer technology also allows students to learn by discovering facts independently through practical and powerful activities that endorse cognitive development and autonomous learning. Furthermore, it provides students with the freedom and opportunities to interact with complex mathematical objects (Nicaise & Barnes, 1996:210), facilitates students' ability to self-regulate (Nicaise & Barnes, 1996:210), affects students' attitudes positively (Dunham & Dick, 1994:443) and improves students' problem-solving skills (Dunham & Dick, 1994:443).

The literature also revealed that a technologically rich classroom provides a good learning environment in which students are actively involved, share and participate in the learning of mathematics and work collaboratively. Farrell (in Dunham and Dick, 1994:443) notes that students became more active in classrooms in which graphing technology was being used, with more group work, investigation and exploration and real problem-solving. In a technologically rich mathematics classroom, students have the opportunity to experiment and find out for themselves (Pollak, 1986:351) and their roles can undergo a shift from passive receivers of information to becoming more involved in group work, real problem-solving, investigating, symbolising and consulting with technology (Farrell in Dunham & Dick, 1994:443). Nicaise and Barnes (1996:208) assert that technological tools allow students "to observe and interact with individuals, many of whom have divergent views and opinions." According to Nicaise and Barnes students' interaction "provide students with opportunities to react to differing views, challenge other beliefs, and reflect their own ideas" (Nicaise & Barnes, 1996:208). Rich, Simonsen, Beckmann and Davis (in Dunham and Dick, 1994:443) also report a shift to fewer lectures by teachers and more investigation by students in technologically rich classrooms. Furthermore, Nicaise and Barnes (1996:210) points out that during the use of technology the role of the teacher shifted from information provider to guide, scaffolder and problem or task presenter. Heid (1997:24) also reports that in using technological tools "…there was less teacher control of the classroom activities and those teachers were less likely to function as authoritative experts and more likely serve as collaborators."

Although most of the findings from the literature indicated that technological tools such as computers and calculators have a positive impact on students' learning of mathematics, the findings of some researchers were not positive. Hall (1993), Pankow (1994), Rich (1993), Ritz (1999) and Smith (1996) reported that there were no significant differences in achievement between students who used technology (graphic calculator) and students who did not use it. Becker (in Dunham and Dick, 1994:442) found that the use of graphing technology did not improve students' understanding of functions in a college pre-calculus course. Giamati (1991) also reported that the use of technology (graphic calculator) affected students' performance negatively.

5.2.2 Summary of the Findings of the Empirical Investigation

The findings from the pre-test (task 1) (see table 4.3) showed that the mean performance of the experimental group wasn't significantly different from the mean performance of the control group. This indicated that the two groups were comparable before the experiment started in terms of their understanding of functions as measured by a performance test.

The findings from the post-test (task 2) (see table 4.6) showed that the mean performance of the experimental group was significantly higher than the mean performance of the control group. This indicated that the use of computers had a positive impact on students' mastering of quadratic functions concepts.

Furthermore, the students' responses to the questionnaire (see tables 4.9 and 4.10 and figures 4.1 and figure 4.2) as well as the interviews showed that the use of computers can positively influence students' learning (understanding) of functions in terms of problem-solving. More specific, the use of computers can:

- allow students to draw graphs of functions quickly and easily;
- encourage students to engage with and explore the nature and properties of quadratic functions and their graphs actively;
- motivate students to learn mathematics (quadratic functions);
- create positive attitudes in students towards quadratic functions in particular and towards mathematics in general;
- encourage students to work in a group;
- facilitate students to develop self-regulation (self-observation, self-evaluation and self-reaction); and
- encourage students to interact among themselves and with their teacher.

However, the use of the MS Excel and RJS Graph software can have some constraints in learning functions because these software do not show all the processes (algorithmic computations and plotting points) involved in drawing graphs in the same way as teaching using a blackboard. Nevertheless, these constraints might be resolved by providing students with more powerful software.

5.3 Limitations of the Study

The small sample size and lack of random assignment of subjects to the groups were limitations. Besides, the experimental period was rather short.

There were also some other problems:

- Some of the students in the experimental group had poor computer skills and I had to teach them basic computer skills in extra free times.
- There were electricity interruptions during the experiment and I had to arrange make-up classes.

5.4 Conclusion

The aim of the empirical investigation was to investigate what influence the use of computers using MS Excel and RJS Graph software has on grade 11 Eritrean students' understanding of functions in the learning of mathematics. The results of this investigation (see sections 4.3, 4.4 and 4.5) indicated that the use of computers has a positive impact on students' achievement, problem-solving skills or exploration of mathematical ideas, motivation, attitude and the classroom environment which are similar to the findings reported in the literature (see sections 2.4.1, 2.4.2 and 2.4.3). Students can analyze functions and their graphs quickly, represent functions in different ways and solve real life problems using computers. Students can be encouraged to explore the nature and properties of functions and their graphs on their own, work in a group, discuss concepts, make conjectures and verify their findings using computers. Thus, if provided with computers, students can learn functions and their graphs through constructivist methods better than the traditional paper-pencil way of teaching and learning functions.

5.5 Recommendations

It is recommended that:

- More studies should be done to investigate what influence the use of computers has on Eritrean students' learning of functions and other mathematical concepts across all grades in the secondary school.

- School mathematics curriculum designers and teachers should be made aware of the role and influence of the use of computers in mathematics (functions) instruction so that students can improve their mastery of mathematical concepts.

- A majority of Eritrean mathematics teachers are not trained to use computers in their teaching and assessment. Thus, they need to be trained to use computers as tools in mathematics classrooms in order to have confidence in incorporating computers into their mathematics programs.

Bibliography

Alexander, M. P. 1993. The effective use of computers and graphing in college algebra. *Dissertation Abstracts International*, 54(6): 2080A.

Arcavi, A. 1995. Teaching and learning algebra: Past, present and future. *Journal of Mathematical Behavior*, 14(1): 145-162.

Ary, D., Jacobs, L. C. & Razavieh, A. 1990. *Introduction to research in education. 4th edition*. Orlando, Florida: Holt, Rinehart and Winston.

Bailey, K. D. 1994. *Methods of social research. 4th edition*. New York: MacMillan.

Battista, M. T. 2001. Shape makers: A computer environment that engenders students' construction of geometric ideas and reasoning. *Computers in the Schools*, 17(1-2): 105-120.

Beckmann, C. E., Senk, S. L. & Thompson, D. R. 1999. Assessing students' understanding of functions in a graphic calculator environment. *School Science and Mathematics*, 99(8): 451-456.

Best, J. W. 1977. *Research in education. 3rd edition.* New Jersey: Prentice-Hall Inc.

Bezuidenhout, J. 1998. First-year University students' understanding of rate of change. *International Journal of Mathematical Education in Science and Technology*, 29(3): 389-399.

Bless, C. & Highson-Smith, C. 2000. *Fundamentals of social research methods: An African perspective.* Cape Town: Juta Education Pty Ltd.

Bonk, C. J. & King, K. S. 1998. *Electronic collaborators*. London: Lawrence Erlbaum Associates.

Branca, N. A., Breedlove, B. A. & King, B. 1992. Calculators in the middle grades: Access to rich mathematics. In J. T. Fey (Ed.), *Calculators in Mathematics Education*. Reston: The NCTM: 9-13.

Casey, J. M. 1997. *Early literacy: The empowerment of technology.* Englewood: Libraries Unlimited.

Cave, R. C. 1995. Graphing, bit by bit. *The Mathematics Teacher*, 88(5): 372-373, 431.

Chandler, P. A. 1993. The effect of the graphing calculator on high school students' mathematical achievement. *Dissertation abstracts international*, 53 (11): 3832A.

Cobb, P., Yackel, E. & Wood, T. 1992. A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, 23(1): 2-33.

Cohen, L. & Manion, L. 1980. *Research methods in education*. London: Routledge.

Collins, A. 1991. The role of computer technology in restructuring schools. *Phi Delta Kappan,* 73(1): 28-36.

Confrey, J. 1992. Using computers to promote students' inventions on the function concepts. In S. Malcom, L. Roberts & K. Sheingold (Eds.), *This year in school science 1991*. Washington, DC: American Association for the Advancement of Science: 141-174.

Connell, M. L. 2001. Actions on objects: A metaphor for technology-enhanced mathematics instruction. *Computers in the Schools*, 17(1-2): 143-172.

Cuoco, A. A. & Goldenberg, E. P 1996. A role for technology in mathematics education. *Journal of Education, 178*(2): 15-31.

Davidenko, S. 1997. Building the concept of function from students' everyday activities. *The Mathematics Teacher*, 90(2): 145-149.

De Villiers, M. D. 1999. *Rethinking proof with the Geometer's Sketchpad.* Berkeley, CA: Key Curriculum Press.

De Villiers, M. D. 2004. Using dynamic geometry to expand mathematics teachers' understanding of proof. *International Journal of Mathematics Education, Science and Technology, 35(5): 703-724.*

De Vos, A. S (Ed.). 2002. *Research at grass roots: For the social sciences and human service professions. 2nd edition*. Pretoria: Van Schaik publishers.

Denzin, A. K. & Lincoln, Y. 2000. *Handbook of qualitative research*. Thousand Oaks: Sage publications Inc.

Dessart, D. J., DeRidder, C. M. & Ellington, A. J. 1999. The research backs calculators. *Mathematics Education Dialogue, 2*(3): 6.

Dreiling, K. M. 2007. *Graphing calculator use by high school mathematics teachers of Western Kansas*. PhD Dissertation. Manhattan, Kansas : Kansas State University.

Dugdale, S. 1993. Visualizing Polynomial Functions. In L. Lum (Ed.), *Proceedings of the fourth international conference on technology in collegiate mathematics*. Massachusetts: Addison-Wesley Publishing Company: 102-108.

Dunham, P. H. 1993. Teaching with graphing calculators: A survey of research on graphing technology. In L. Lum (Ed.), *Proceedings of the fourth international conference on technology in collegiate mathematics*. Massachusetts: Addison-Wesley Publishing Company: 89-101.

Dunham, P.H. & Dick, T. P. 1994. Research on graphing calculators. *The Mathematics Teacher*, 87(6): 440-445.

Durmus, S. 2000. The effects of the use of technology on college algebra students' achievement and attitudes towards mathematics: A constructivist approach. *Dissertation of Abstracts International*, 60(10): 3622A.

Elia, I. & Spyrou, P. 2006. How students conceive function: A trairchic conceptual-semiotic model of the understanding of a complex concept. *The Montana Mathematics Enthusiast,* 3(2): 256-272.

Erickson, D. K. 1999. A problem-based approach to mathematics instruction. *The Mathematics Teacher*, 92 (6): 516-521.

Ernest, P. 1989. What's the use of LOGO? In P. Ernest (Ed.), *Mathematics teaching: The state of art.* London: The Falmer Press: 33-44.

Ernest, P. 1991. *The Philosophy of Mathematics Education.* Hampshire: The Falmer Press.

Fey, J.T. 1989. Technology and mathematics education: A survey of recent developments and important problems. *Educational Studies in Mathematics,* 20(3): 237- 272.

Funkhouser, C. 1993. The influence of problem solving software on student attitudes about mathematics. *Journal of Research on Computing in Education*, *25(*3): 339-346.

Gall, D.M., Borg, R.W. & Gall, P. J. 1996. *Educational research: An introduction. 6th edition.* USA: Longman publishers.

Gay, L.R. 1992. *Educational research - Competencies for analysis and application*. Singapore: Merril, MacMillan.

Giamati, C.M. 1991. The effect of graphing calculator use on students' understanding of variations on a family of equations and the transformations of their graphs. *Dissertation Abstracts International*, 52(1): 103A.

Glasersfeld, E.V. 1992. Questions and answers about radical constructivism. In M. K. Pearsall (Ed.), *Scope, sequence and coordination of secondary schools science.* Washington DC: NSTA: 169-182.

Goldenberg, E.P. 2000. *Issues in mathematics education. Thinking (and talking) about technology in mathematics classrooms.* USA: Education Development Centre Inc.

Graham, T. M. and Thomas, O. J. 2000. Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics*, 41(3): 265-282.

Gray, E. M. & Tall, D. O. 1994. Duality, ambiguity and flexibility: A perceptual view of simple arithmetic. *Journal for Research in Mathematics Education*, 26(2): 116-140.

Hall, M. K. 1993. Impact on the graphing calculator on the instruction of trigonometric functions in pre-calculus classes. *Dissertations Abstracts International*, 54(2): 451A.

Hancock, V. 1997. Creating the information age school. *Educational Leadership*, *55*(3): 60-63.

Hannafin, R. D. & Scott, B. N. 2001. Teaching and learning with dynamic geometry programs in student-centered learning environments: A mixed method inquiry. *Computers in the Schools*, 17(1-2): 121-142.

Harvey, J.G. & Osborne, A. 1991. A calculator and computer technologies in mathematics classrooms: A user's guide. In F. Demana, B.K. Waits and J. Harvey (Eds.), *Proceedings of the second annual conference on technology in collegiate mathematics: Teaching and learning with technology*. Reading, MA: Addison-Wesley: 74-86.

Haugland, S. W. & Wright, J. L. 1997. *Young children and technology: A world of discovery.* Massachusetts: Ally and Bacon.

Heid, M. K. 1997. The technological revolution and the reform of school mathematics. *American Journal of Education*, 106(1): 5-61.

Heid, M. K. 1998. *The impact of computing technology on the teaching and learning of mathematics at the secondary level: Implications for Standards 2000*. Accessed on May 25, 2006 from http://mathforum.org/technology/papers/papers/heid/heid.html

Hennessy, S., Fung, P. & Scanlon, E. 2001. The role of graphic calculator in mediating graphing activity. *International Journal of Mathematical Education in Science and Technology*, 32(2): 267-290.

Hiebert, J. Carpenter, T. P., Fennema, E., Fuson, K., Human, P. G., Murray, H. Olivier, I. A. & Wearne, D. 1996. Problem solving as a basis for reform in curriculum and instruction: The case for mathematics. *Educational researcher*. May 1996:12-21.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K., Human, P. G., Murray, H., Olivier, A. I. & Wearne, D. 1997. *Making sense: Teaching and learning mathematics with understanding*. Portsmouth: NH Heinemann.

Hitchcock, G. & Hughes, D. 1995. *Research and the teacher: A qualitative introduction to school-based research.* London: Routledge.

Hollar, J. & Norwood, K. 1999. The effects of a graphing-approach intermediate algebra curriculum on students' understanding of function. *Journal for Research in Mathematics Education,* 30(2): 220-226.

Ibrahim, A. 2004. Computer-supported collaborative problem solving and anchored instruction in a mathematics classroom: An exploratory study. *International Journal of Learning Technology*, 1(1): 16-39.

Ijeh, S. B. 2003. *The effect of scaling in the understanding of algebraic graphs for grade 9 learners.* MEd dissertation. Pretoria: UNISA.

Jick, T. D. 1994. *Mixing qualitative and quantitative methods: Triangulation in action*. *Comparative Education in Qualitative Research in Education* Vol. 17. Pretoria: UNISA.

Johnson, B. & Christensen, L. 2000. *Educational research - Quantitative and qualitative approaches*. Boston: Aylln and Bacon.

Jones, D. 1997. A conceptual framework for studying the relevance of context to mathematics teachers' change. In E. Fennema and S. Nelson (Eds.), *Mathematics teachers in transition*. New Jersey: Lawrence Erlbaum Associates Publishers: 131-154.

Kamii, C. 1994. *Young children continue to reinvent arithmetic-3rd grade implications of Piaget's theory*. New York: Columbia University, Teachers college Press.

Kanselaar, G. 2002. *Constructivism and socio-constructivism*. Accessed on May 06, 2006 from http://edu.fss.uu.nl/medewerkers/gk/files/Constructivism-gk.pdf

Kinzer, C. K. 1986. A 5 part categorization for use of microcomputers in reading classrooms. *Journal of Reading*, 30(3): 226-232.

Kulik, C. C. & Kulik, J. A. 1991. Effectiveness of computer-based instruction: An updated analysis. *Computers in Human Behavior*, *7(2):* 75-94.

La Torre, D. 1993. The impact of super calculators on the teaching and learning of linear algebra. In L. Lum (Ed.), *Proceedings of the fourth international conference on technology in collegiate mathematics*. Massachusetts: Addison-Wesley Publishing Company: 162–168.

Laughbaum, E. D. 1998. Testing students with hand-held technology. In E. D. Laughbaum(Ed.), *Hand-held technology in mathematics and science education: A collection of papers*. Columbus: The Ohio State University: 184-190.

Leinhardt, G., Zaslavsky, O. & Stein, M. K. 1990. Functions, graphs and graphing: Tasks, learning and teaching. *Review of Educational Research*, 60(1): 1-64.

Lewis, R. 1999. The role of technology in learning: Managing to achieve a vision. *British Journal of Educational Technology, 30 (2): 141-150.*

Liu, L. & Cummings, R. 2001. A learning model that stimulates geometric thinking through use of PCLogo and Geometer's Sketchpad. *Computers in the Schools*, 17(1-2): 85-104.

Makgato, M. 2003. *The development of a curriculum for technology teacher education and training: A critical analysis*. DEd dissertation. Pretoria: UNISA.

Matusevich, M. N. 1995. *School reform: What role can technology play in a constructivist setting?* Accessed on June 05, 2006 from http://delta.cs.vt.edu/edu/fis/techconf.html

Means, B. & Olson, K. 1995. *Technology's role in education reform (SRI Project 2882):* SRI International.

Ministry of Education. 2005. *Curriculum framework for secondary school mathematics*. Unpublished Document. Asmara: Ministry of Education.

Morrelli, R. 1990. The student as knowledge engineer: A constructivist model for science education. *Journal of Computing in Higher Education, 2*(1): 78-102.

Murray, H., Olivier, A. & Human, P. 1993. Learning through problem solving and social interaction. *Proceedings of the 17th international conference for the psychology of mathematics education*. Tsukuba, Japan, July 1993:193-203.

Nakin, J. N. 2003. *Creativity and divergent thinking in geometry education.* DEd Dissertation. Pretoria: UNISA.

National Council of Teachers of Mathematics (NCTM). 1989. *The curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). 1995. *Algebra in a technological world: Addenda series grades 9 - 12.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). 2003. *The use of technology in the learning and teaching of mathematics* (Position statements). Reston, VA: NCTM. Accessed on June 12, 2006 from http://www.nctm.org/about/content.aspx?id=6360

Ndlovu, M. 2004. *An analysis of teacher competencies in a problem-centred approach to dynamic geometry teaching*. MEd Dissertation. Pretoria: UNISA.

Neuman, W. L. 1997. *Social research methods: Qualitative and quantitative approaches.* Boston: Aylln and Bacon.

Newby, T. J., Stepich, J. D. & Russel, J. D. 2000. *Instructional technology for teaching and learning: Designing instruction, integrating computers and using media.* Ohio: Merill.

Nicaise, M. & Barnes, D. 1996. The union of technology, constructivism and teacher education. *Journal of Teacher Education*, 470(3): 205-212.

Norman, F. A. & Prichard, M. K. 1992. Technology in pre-service teacher education programs for elementary and secondary mathematics teachers. In J. G. Harvey, F. D. Demana and B. K. Waits (Eds.), *Proceedings of the third international conference on technology in collegiate mathematics.* Massachusetts: Addison-Wesley Publishing Company: 259-261.

Ntsohi, M. E. 2005. *An investigation into the problems encountered by learners and teachers of grade 9 algebra on understanding linear equations: A critical analysis.* MEd Dissertation. Pretoria: UNISA.

Pankow, C. K. 1994. The effects of college student use of computers the learning of algebraic concepts. *Dissertation Abstracts International*, 55(12): 27774A.

Pollak, H. O. 1986. The effect of technology on mathematics curriculum. In M. Carss (Ed.), *Proceedings of the fifth International congress on mathematical education.* Boston: Birkhauser Press: 346-351.

Post, T. 1980. The role of manipulative materials in the learning of mathematical concepts. In M. M. Lindquist (Ed.), *Selected issues in mathematics education*. USA: McCutchan Publishing Corporation: 109-131.

Posthuma, A. B. 2000. *Introduction to pre-calculus A: Only study guide for FDEME1-J · MPE110-6 · MSE110-N.* Pretoria: UNISA.

President's Committee of Advisors on Science and Technology. 1997. *Report to the President on the use of technology to strengthen K-12 education in the United States* (President Report). Washington, DC: PCAST. Accessed on February 22, 2007 from http://www.ostp.gov/PCAST/k-12ed.html

Pugalee, D. K. 2001. Algebra for all: The role of technology and constructivism in an algebra course for at-risk students. *Preventing School Failure*, 45(4): 171-177.

Quesada, A. R. and Maxwell, M. E. 1994. The effects of using graphing calculators to enhance college students' performance in pre-calculus. *Educational Studies in Mathematics*, 27(2): 205-215.

Reys, R. E., Suydam, M. N., Lindquist, M. M. & Smith, N. L., 1998. *Helping children learn mathematics. 5th edition.* Needham Heights, MA: Allyn and Bacon.

Rich, B. S. 1993. The effect of the use of graphing calculators on learning the function concepts in pre-calculus mathematics. In L. Lum (Ed.), *Proceedings of the fourth international conference on technology in collegiate mathematics*. Massachusetts: Addison-Wesley Publishing Company: 389-393.

Ritz, C. T III. 1999. The effect of using graphing calculators on student achievement in introductory algebra and geometry. *Dissertation Abstracts International, 38*(3): 51.

Roth, W. F. 1999. Computers can individualize learning AND raise group-interaction skills. *The Educational Digest (November, 1999).*

Ruthven, K. 1992. Personal technology and classroom change: A British perspective. In J. T. Fey (Ed.), *Calculators in Mathematics Education.* Reston: The NCTM: 91-100.

Sax, G. 1979. *Foundations of educational research*. Englewood Cliffs, NJ: Prentice-Hall.

Shuard, H. 1992. CAN: Calculator use in the primary grades in England and Wales. In J. T. Fey (Ed.), *Calculators in Mathematics Education*. Reston, VA: The NCTM: 33-45.

Sivin-Kachela, J. & Bialo, E. R. 1996. *Report on the effectiveness of technology in schools.* Washington, DC: Software Publishers Association.

Smith, B. A. 1996. A meta-analysis of outcomes from the use of calculators in mathematics education. *Dissertation Abstracts International, 58*(3A): 125.

Smith, K. B. & Shotsberger, P. G. 1997. Assessing the use of graphing calculators in college algebra: Reflecting on dimensions of teaching and learning. *School Science and Mathematics, 97*(7): 368-376.

Stein, M. K., Silver, E. A. & Smith, M. S. 1998. Mathematics reform and teacher development from a community of practice perspective. In J. Greeno & S. Goldman (Eds.), *Thinking practices: A symposium on mathematics and science learning.* Hillsdale, NJ: Erlbaum: 17-52.

Sutherland, R. 1990. The changing role of algebra in school mathematics: the potential of computer-based environment. In P. Dowling and R. Noss (Eds.), *Mathematics versus the national curriculum*. London: Falmer Press: 154-175.

Tall, D. & Thomas, M. 1991. Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics,* 22(2): 125-147.

Taplin, M. 2007. *Mathematics through problem solving.* Accessed on January 03, 2007 from http://www.mathgoodies.com/articles/problem_solving.html

Taylor, R. 1980. Computers and calculators in the mathematics classroom. In M. M. Lindquist (Ed.), *Selected issues in mathematics education*. USA: McCutchan Publishing Corporation: 132-158.

Thanasoulas, D. 2002. *Constructivist learning.* Accessed on May 10, 2006 from http://www3.telus.net/linguisticsissues/constructivist.html

The International Study Center. 1998. *International study finds the Netherlands and Sweden best in mathematics and science literacy.* Accessed on June 10, 2006 from http://timss.bc.edu/timss1995i/presspop3.html

Tooke, D. J. 2001. Mathematics, the Computer and the Impact on Mathematics Education. *Computers in the Schools*, 17(1-2): 1-8.

Waits, B. K. & Demana, F. 2000. Calculators in mathematics teaching and learning: Past, present and future. In M. J. Burke & F. R. Curcio (Eds.), *Learning mathematics for a new century.* Reston, VA: NCTM: 51-66.

Wheatley, G. H & Shumway, R. 1992. The potential of calculators to transform elementary school mathematics. In J. T. Fey (Ed.), *Calculators in Mathematics Education*. Reston: The NCTM: 1-8.

Wiest, L. R. 2001. The role of computers in mathematics teaching and learning. *Computers in the Schools*, 17(1-2): 41-56.

Yerushalmy, M. & Gilead, S. 1997. Solving equations in a technological environment. *The Mathematics Teacher*, 90(2): 156-162.

Zaslavsky, O. 1997. Conceptual obstacles in the learning of quadratic functions. *Focus on Learning Problems in Mathematics,* 19(1): 20-44.

APPENDIX A

Pre-test (Task 1)

Instruction: Answer all questions clearly (show all your works).

1. For each of the following sets,
   a) State the domain and range. (3 points)
   b) Tell whether or not the set represents a function. (3 points)

   i. {(3, 4), (4, 5), (6, 7), (7, 3)}
   ii. {(8, 2), (8, 3), (-1, 2)}
   iii. {(x, 12): x is whole number}

2. Given the domain {-2, -1, 0, 1, 2}. Find the range corresponding to each of the following functions.
   a) $f(x) = x^2 - 16$ (2 points)
   b) $f(x) = 2x^2 + 5x + 3$ (2 points)

3. For each of the following quadratic functions,
   a) Find the x and y - intercepts of the functions. (9 points)
   b) Find the domain and the range of the functions. (6 points)
   c) Sketch the graph of the functions. (12 points)
   d) Find the roots (or zeroes) of the functions. (3 points)
   e) Find the maximum or minimum value of the functions. (6 points)

   a. $f(x) = x^2 - 6x + 5$
   b. $f(x) = 2x^2 - 8$
   c. $g(x) = -x^2 -2 x + 3$
   d. $g(x) = 8x - x^2$
   e. $h(x) = 4x^2 + 4$

f. $h(x) = -2x^2 - 2x - 3$

4. Make a general statement about the graph of $f(x) = ax^2 + bx + c$ when "*a*" is *positive* and when "*a*" is *negative.* (4 points)

APPENDIX B

Post-test (Task 2)

Instruction:

Answer all questions clearly (show all your works).

1. In a business, the relation between sales and time is given in a chart (table).

Hours	Sales
1	15
2	25
3	32

a) Is the relation a function? Why or why not? (3 points)
b) What is the domain of the relation? (1 point)
c) What is the range of the relation? (1point)

2. When you study the swing of a pendulum, you will notice that the time, T, in seconds taken for one complete swing of a pendulum is less if the length, L, in centimeters of the pendulum is shortened. The relationship between T and L is given by

$$T = 2\pi \frac{L}{980}$$

The length, L, in centimeters of a pendulum is related to the temperature by the following relationship.

$L = 50 + 0.0035C$, where C is the temperature, in Celsius

Tacking the above into considerations, answer the following questions:

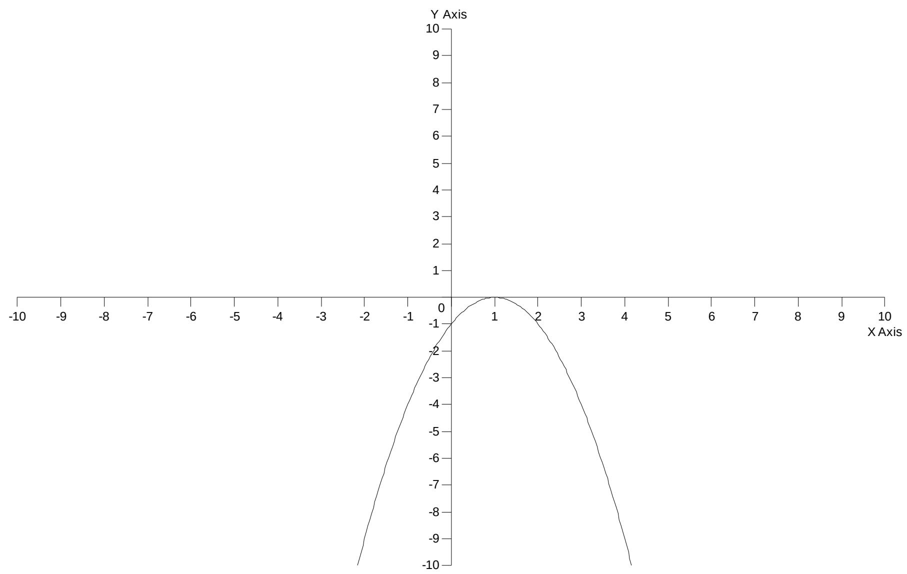
a) Express period, T, as a function of C. (3 points)
b) Calculate the period, T, of the pendulum if the temperature is 0^oC. ($\pi \approx 3.14$) (1 point)
c) Calculate the period, T, of the pendulum if the temperature is 20^oC. ($\pi \approx 3.14$) (1 point)
d) How has the period of the pendulum been affected by the change in temperature? (2 points)

3. For the following quadratic functions:

i) $f(x) = x^2 + 2x - 3$
ii) $g(x) = -x^2 - 5x - 6$
iii) $q(x) = -3x^2 - 12$

a) Find the x and y - intercepts of the functions. (6 points)
b) Find the domain and range of the functions. (6 points)
c) Sketch the graphs of the functions. (12 points)
d) For what values of x are the functions increasing? And for what value of x are the functions decreasing? (6 points)
e) Find the maximum or minimum value of the functions. (3 points)

4. The sketch represents the graph of $f(x) = ax^2 + bx + c$. Which one the following is **true**. (2 points)



a) a = 0
b) a > 0
c) a < 0
d) All are true

5. From the top of a building 75 m high, a ball is dropped. The height of the ball at time, t (in seconds) is given by $h = A - 4.9t^2$, where A is the height of the ball before it is dropped.

a) Draw a graph of the function. (4 points)

b) What is the height of the ball after 3 seconds have elapsed? (1 point)

c) Will the ball strike the ground after 4 seconds? Why? (3 points)

6. During a stunt, the power dive of a plane is given by the equation, $h = t^2 - 10t + 80$, where h (in meters) is the height of the plane after time, t (in seconds).

a) Draw a graph of the path of the plane. (4 points)

b) How high is the plane at the start of the dive (i.e. at t = 0)? (1 point)

c) How high above the ground level is the plane at its minimum point? (2 points)

7. A rectangular field is to be enclosed with 800 m of fencing. What is the maximum area possible? (8 points)

APPENDIX C

Questionnaire

Instructions:

- *Answer all questions.*
- *All answers will be treated in the strictest confidence.*
- *Circle your choice from the given alternatives (indicate the extent to which you agree or disagree with the following statements).*

1. Quadratic functions is an interesting topic to learn using a computer (MS Excel and RJS Graph software).
   a) Strongly disagree
   b) Disagree
   c) Agree
   d) Strongly agree
2. It is convenient to solve a quadratic function problem using a graph if you use a computer (MS Excel and RJS Graph software).
   a) Strongly disagree
   b) Disagree
   c) Agree
   d) Strongly agree
3. The use of a computer (MS Excel software) in learning quadratic functions enables one to create tables of values of the functions quickly.
   a) Strongly disagree
   b) Disagree
   c) Agree

d) Strongly agree

4. The use of a computer (RJS software) in learning quadratic functions enables one to draw graphs of the functions easily.
    a) Strongly disagree
    b) Disagree
    c) Agree
    d) Strongly agree
5. The use of a computer (MS Excel and RJS Graph software) in learning quadratic functions enables one to get sufficient time to investigate the nature and properties of the functions and their graphs.
    a) Strongly disagree
    b) Disagree
    c) Agree
    d) Strongly agree
6. The use of computers in learning quadratic functions gives students the opportunity to work in a group.
    a) Strongly disagree
    b) Disagree
    c) Agree
    d) Strongly agree
7. The use of computers in learning quadratic functions gives students the opportunity to share views among themselves.
    a) Strongly disagree
    b) Disagree
    c) Agree
    d) Strongly agree
8. The use of computers in learning quadratic functions gives students the opportunity to share views with their teacher.
    a) Strongly disagree
    b) Disagree
    c) Agree

d) Strongly agree

9. The use of a computer in learning quadratic functions motivates a student to work his/her own problems.
    a) Strongly disagree
    b) Disagree
    c) Agree
    d) Strongly agree
10. The use of computers in learning quadratic functions gives students the opportunity to engage with real life mathematical problems.
    a) Strongly disagree
    b) Disagree
    c) Agree
    d) Strongly agree

APPENDIX D

Questions for an Interview

1. How are you being taught mathematics?
2. What were your experiences in learning quadratic functions (using paper and pencil)?
3. What can you say about the importance of computers (MS Excel and RJS Graph software) in learning quadratic functions?
4. What were your experiences in learning quadratic functions using computers using MS Excel and RJS Graph software?
5. Do you have anything else to add?

Thank you!